Analysis of the Abstraction Process of Continuity Knowledge

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Abstract

This study aimed to analyze the abstraction process of twelve-grade students' continuity knowledge through the RBC+C abstraction model. With this aim, a semi-constructed interview was conducted with two twelfth-grade students and recorded with a video camera. Two different research problems were addressed in the interview, and the students were asked to think about these problems loudly by discussing each other. As a result, it was understood that these students found difficult to build-with some prior knowledge that is required for abstracting continuity knowledge even though they recognized them. Nevertheless, it was observed that the students constructed continuity knowledge to some extent.

Keywords: abstraction, cognitive actions, continuity, high school students, RBC+C

1. Introduction

1.1 Introduce the Problem

Continuity is one of the most important and fundamental subjects of mathematics (Denbel, 2014; Duru, Köklü, & Jakubowski, 2010; Jayakody & Zazkis, 2015; Petrunić, 2009). It is also a concept encountered in daily life (The research questions of this study are some examples of the subject use in daily life). Although it may seem intuitively easy to understand, students have various misconceptions about this subject (Amatangelo, 2013; Bezuidenhout, 2001; Duru, Köklü, & Jakubowski, 2010). Continuity in daily language means continuing without interruption. However, continuity as a mathematical concept encompasses not only the meaning in everyday language but also situations that are contrary to this meaning (Aydın & Kutluca, 2010). Even the daily meaning evoked by the subject could cause difficulties in comprehending continuity. The research conducted in this area (Baştürk & Dönmez, 2011; Bridgers, 2007; Durmuş, 2004) also confirms this. Additionally, the concepts of limit and continuity are also effective in learning the concept of derivative (Yetim, 2004). Therefore, considering the relevance of continuity and limit concepts to further topics (such as derivatives and integral), it is vital to learn the concept of continuity thoroughly and to investigate how this is done.

Knowledge generation begins with the need for a new structure and includes the generation of an abstracted entity (Dreyfus, Hershkowitz, & Schwarz, 2001; Hershkowitz, Hadas, Dreyfus, & Schwarz, 2007; Hershkowitz, Schwarz, & Dreyfus, 2001; Monaghan & Ozmantar, 2006; Tsamir & Dreyfus, 2002). In the process of the emergence of action, individuals need the information they already know in solving a problem and apply the prior knowledge to the abstraction process of the new knowledge (Dreyfus, 2007; Dreyfus, Hershkowitz, & Schwarz, 2001). Different abstraction theories have been introduced for centuries to examine students' knowledge generation process. One of the theories is the Recognizing(R) Building-with(B) Construction(C) + Consolidation(C) abstraction model introduced by Hershkowitz, Schwarz, and Dreyfus (2001), which provides an opportunity to study the abstraction process of knowledge through different cognitive actions (recognizing, building-with, construction, consolidation). According to the RBC+C model, the learner needs to be familiar with the old structures to reach a new structure, establish connections and reveal relationships (Sezgin-Memnun, Aydın, Özbilen, & Erdoğan, 2017). These old structures, connections, and relationships could be used to construct a new structure. Then, one reinforces the process to make the abstraction meaningful. *Recognizing*, which is one of the cognitive actions in the RBC+C abstraction model, refers to attribute a meaning to the new situation by utilizing the prior knowledge of the individual in the learning process, and *building-with* includes reaching the results by using the prior knowledge in the generation of the new knowledge (Hassan & Mitchelmore, 2006; Hershkowitz, Schwarz, & Dreyfus, 2001). After recognizing and building-with actions, *construction* may take place. This action involves reconstructing of students' knowledge and the generation of new knowledge (Dreyfus, 2007). *Consolidation* action requires reinforcement of the newly constructed knowledge (Dreyfus, Hershkowitz, & Schwarz, 2001; Hershkowitz, Schwarz, & Dreyfus, 2001). Consolidation is not directly involved in the process of generating knowledge, but instead, it refers to the stage after the knowledge is generated. Since this model enables to examine the abstraction process of mathematical knowledge through observable actions (Dreyfus, 2007), this abstraction model was used in this study as a tool for the analysis of the abstraction process of continuity knowledge.

1.2 Explore Importance of the Problem

It is thought that this study will provide clues for teachers about the understandings of their students in terms of continuity and how students can achieve better learning. With this study, it is believed that teachers will be able to see where and how they should direct their students and realize what points their students experience difficulties in the concept of continuity. In this study unlike other studies in the academic literature, it is aimed to investigate twelfth-grade students' abstraction process of continuity knowledge employing the RBC+C abstraction model, which provides an opportunity to analyze through cognitive actions. The most important feature that distinguishes this research from others is that this study was carried out with technical high school students. As can be understood from the application questions in the study, the effect of the continuity concept on the area where the technical high school students are educated makes them the relevant research sample on this subject. For this purpose, the research question was as follows: How is twelfth-grade students' abstraction process of continuity knowledge?

This research is important in terms of allowing to investigate twelfth-grade students' learning processes of continuity and examining the process of knowledge generation by twelfth graders through cognitive actions, namely *recognizing, building-with, construction*, and *consolidation*. It is thought that this study will contribute to the field with these aspects.

1.3 Describe Relevant Scholarship

The concept of continuity is based on the concept of limit; therefore, learning the limit concept becomes a requirement before learning the continuity concept (Mastorides & Zachariades, 2004). Thus, continuity and limit are considered together in some of the research in this field (Amatangelo, 2013; Areaya & Sidelil, 2012; Aydın & Kutluca, 2010; Baştürk & Dönmez, 2011; Takači, Pešić, & Tatar, 2006; Zvichapera, 2016). Amongst these studies, Amatangelo (2013) investigated the conceptions of students related to limit and continuity. The researcher pointed out that *the defined means continuous* as a potentially problematic conception was the most common conception about the concept of continuity. Students defended this conception as "the rule of continuity" without adding the limit value which is supposed to be equal to the defined function value. In addition, some students had conceptions under the titles; the limit means continuous and the limit means discontinuous. Areaya and Sidelil (2012) conducted a study to investigate the difficulties, challenges, and misconceptions of high school students in terms of continuity. The researchers indicated that the lack of ability to draw graphs of functions affects students' understanding of continuity. In addition, they stated that algebraic manipulation and the concept of asymptote have an effect on students' conceptions about continuity. The students in the study also reflected discontinuity at a point as a hole. As a result of the study carried out by Aydın and Kutluca (2010), it was reported that the students thought that the graph of continuous functions should not contain spaces and should consist of one piece. It is also explained that students had difficulties in structuring even the basic knowledge about this subject. In addition, it was revealed that students who intuitively felt the continuity of the function explained it by taking only the right- and left-limit approaches and did not look at whether the function was defined at that point. In Baştürk and Dönmez's (2011) study, teacher candidates stated that if there is a limit of a function at a point, then it must be defined and continuous at that point and that if a graph of a function does not consist of a single piece, then this function is not continuous.

Takači, Pešić, and Tatar (2006) stating that continuity is likely the most difficult subject in calculus reported that high school students explained continuity in function with continuity in the graph. In other words, students considered the graph of a function to determine continuity rather than limit value and function value. As a result of the research conducted by Bridgers (2007), which investigated the concept of continuity with high school teacher and their students, it was reported that high school teachers, who find continuity as an important concept see continuity as a difficult concept. Also, students were confused about the meaning of continuity and had misconceptions about notions of a defined function, a continuous function and the concept of limit. In their study, students could not associate the concepts of limit and continuity. Zvichapera (2016) investigated the conceptions related to continuity derived from different definitions (informal, semi-formal, and formal) and representations (algebraic, graphical, and verbal) of the concept of continuity. According to the study, students' conceptions related to continuity is mostly based on drawing the function graph without lifting the pen and looking for "jumps" and "holes" on the graph. Some students only looked for the limit to determine continuity while some investigated whether the limit value and the function value at a certain point are equal.

2. Method

2.1 Participant (Subject) Characteristics

The RBC+C model can be used to analyze the knowledge abstraction model in studies that consist of individuals or small groups (Tabach, Rasmussen, Dreyfus, Hershkowitz, 2017). Therefore, this study was carried out with two twelfth-grade students together at the same time. *Criterion sampling method*, which is one of the purposive sampling methods was used to determine the students. Criterion sampling involves selecting all cases in accordance with predetermined criteria (Patton, 1990).

Considering the scope of the research, the significance of pre-learning of participants in generating the continuity knowledge can be seen. For this, in order to reach the continuity knowledge, the participants must know the concept of limit but not the concept of continuity. The interview with these two twelfth-grade students was carried out at a time when limit was completed in the mathematics courses taken by these students, but the attainments of continuity were not yet started. Therefore, two successful twelfth-grade students, who study in a technical Anatolian high school and were confirmed to have the ability to express themselves mathematically by their teachers were selected by taking into account their mathematics course grades. This study has been carried out with these two twelfth-grade students, who were willing to participate in the research.

2.2 Research Instrument

A research form consisting of two different research problems was prepared by making sure that the problems are compatible with daily life and understandable for the purpose of generating the continuity knowledge of these two participating students. Then, in order to determine the suitability of these two problems in terms of examining students' continuity knowledge generation, opinions of a total of six experts in their field were taken, and these problems were rearranged in line with the suggestions by these experts. The researchers made preliminary preparations about the questions that the students would have in addition to the questions prepared in advance. In this context, it has been decided that questions such as "Why have you thought in this way?" or "Can you look through once more?" might be asked for guidance when considered necessary during the interview.

2.3 Research Design

This case study aimed to investigate the process of continuity knowledge generation of twelfth-grade students. A case study is a research method used to understand, define, and describe the causes and consequences of current issues when the researcher has no control over variables (Leymun, Odabaşı, & Yurdakul, 2017). *Semi-structured interview* method was used in the study. Semi-structured interviews are often preferred in academic studies due to the standard and flexibility because it eliminates the limitations of tests and surveys based on writing and filling. It also helps to gain in-depth knowledge of a particular subject (Yıldırım & Şimşek, 2003).

2.4 Data Collection

Before the interview, it was explained that the aim was to analyze the process of reaching an answer rather than finding the correct or an incorrect answer in the research. For this reason, the students were told that they should explain their thoughts in detail and without fear of making mistakes during the interview. After this explanation was made, these two twelfth-grade students who participated in the study were given research problems written on a piece of paper (research form). The students were given pseudonyms; Zehra (Z) was used for one of the students and Simge (S) for the other in the interview transcript. The researcher's statements were also included in the interview transcript with the abbreviation of (A).

One of the researchers (A) participated as an observer in the interview and tried to guide students correctly when needed. The school provided a classroom for the interview, and the interview was carried out in that classroom by two participant students and the observer at the same time. Thus, it was tried to make the students think loudly about the problems by discussing problems with each other. In this way, verbal expressions of the students during the discussions with each other will contribute to the observability of the process. It is also assumed that their dialogues between themselves and the observer will emerge the difficulties in understanding and what they may misunderstand, that is, their mental processes. Also, the interview was recorded by a *video camera* placed where the students can see it.

2.5 Data Analysis

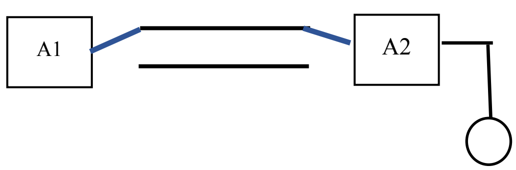
The students' responses in the interview transcript prepared by considering the video recording were analyzed through *descriptive analysis*. First, the two researchers prepared the interview transcript separately by listening to the video recording. Afterward, the transcripts written by the two researchers separately at different times were examined, and in the case of differences, the video recording was listened to again. The interview transcript was finalized according to the re-examination of the video recording. As for the actions of the study, they were defined as cognitive actions in the RBC+C abstraction model, namely *recognizing, building-with, construction*, and *consolidation*. The research findings of the study were attained by considering these four epistemic actions for the analysis and interpretation of the interview transcripts. Within the scope of the study, the practice, which was carried out with two twelfth-grade students, took approximately 35 minutes.

3. Results

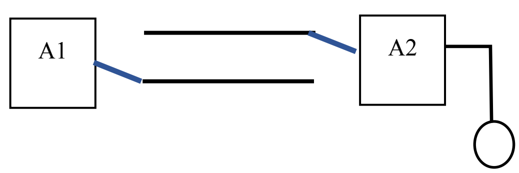
The first problem in the research form was prepared to scrutinize the difference between the two different situations that the students are familiar with their daily lives. At this point, the first question in the research form was prepared by focusing on the meaning of continuity in daily life. So, this problem aims to encourage students to reflect based on the informal definition of continuity. In this context, two different options were provided to the students below. While one of the options includes *continuity*, the other includes *discontinuity*.

The first problem in the research form is: "*Double-switch binding is used in houses at the beginning and end of stairs, which is called 'vavien/two-way-switch/contactor.' You can switch the lights on at the beginning of the stairs and you can switch the lights off at the end of the stairs, and vice versa. For the cases stated below, would lights be on when the switch 1 (A1) is put in `on` position? Why? (A1: Switch 1, A2: Switch 2)*"

(a)



(b)



The students studied the two different conditions in the electrical circuit, which are continuous and discontinuous, for a total of 3 minutes and did not need to ask any questions about the problem. This situation arose due to the fact that both of the students participating in the study are technical high school students and thus they are more acquainted with electrical circuits as compared to students in other types of schools. Some of the interview texts are given below.

Z20: Hmm… this is the light and those are the switches… (Thinks over for a short time) There is an interruption here. Can I ask something? Now then, there is a switch here (referring to the line…) that part of the switch does not exist… We are coming from this side but there is no place to go. For this reason, the light will not be "on" anyway. Because, as I said it, there is a disconnection here.

A21: Yes…

Z22: So, the light would not be "on" because of the disconnection.

(They continue to stare at the second figure.)

S23: There is not disconnection here.

Z24: Therefore, the light will be "on."

In this section of the interview, Zehra vocally shared what she thought with Simge. Both students said that the light in the first circuit would not be "on," but it would be "on" in the second circuit. At this stage, it is thought that word of "disconnection" which is included in the expression of Z20 could be an important analogy in the transition to the meaning of discontinuity. In the piece of transcript given above [S23], Simge has approved Zehra and expressed continuity by saying "there is not disconnection here." This situation suggests that students began to generate continuity knowledge.

Unlike the first problem in the research from, a case from daily life was expressed in the second problem in the research form and students were asked to draw a graph according to the data given. The aim here is to ensure that students construct the concept of continuity and consolidate it. Therefore, it is important to draw the graph correctly, which is desired to be drawn in this problem. In this problem, the students need to do mathematicalization in order to show whether they can reach the formal definition of continuity. In other words, it is analyzed whether these students could establish relationships between concepts by working on symbols and thus whether these students are able to realize the mathematical process. Then, the students were expected to make correct interpretations according to the graph drawn and find the limit values at certain points; this is necessary and important for the transition to the concept of continuity. At the end of this process, the students were expected to reach continuity notion and continuity at a point notion by making generalizations.

The second problem is: "*There are two different types of electricity meter today which vary according to old and new buildings. First one is the mechanical electricity meter and the second one is the electronic triple-tariff counter.*

There is a single tariff for the mechanical electricity meter, and the unit price is the same for the electricity used within 24 hours. According to the national tariff, for 2015 to be valid from 01.01.2015, the unit price for 1 kilowatt/hour is 31 kurus for electricity used per hour (rounded figures are given).

However, there are 3 different tariffs for the electronic triple-tariff meter. Day time, peak load and night time tariffs and periods are different. According to the national tariff for 2015 to be valid from 01.01.2015, the unit price for 1 kilowatt/hour is;

Day time tariff (between 06.00 - 17.00):29 kurus,

Peak load tariff (between 17.00 - 22.00): 45 kurus,

Night time tariff (between 22.00 - 06.00): 17 kurus

Please plot the tariff/time graph for both meters. It should be that person looking at your graph should be able to understand on which tariff he/she is in and in which period." The students studied this problem for a total of 34 minutes.

In this section, the students first read the problem and then discussed with each other whether to draw one or two graphs. At the end of this discussion, the students decided that they need to draw two graphs. Zehra and Simge decided to draw the mechanical electricity meter's graph first and then the electronic triple-tariff counter's graph. During these drawings, Zehra expressed that the unit price was the same on both graphs, but Simge corrected her and explained that there were different tariffs in the second graph. The drawing of the graph performed by Zehra indicates that this student *recognized* the elements of a graph and *built-with* it for the solution. It is also noteworthy that the students designated meter types as the names of the graphs. This situation shows that the students also recognized the concept of graphs and build-with it and oriented towards a detailed and correct solution to the problem. The graphs drawn by the students are seen in the figure below.

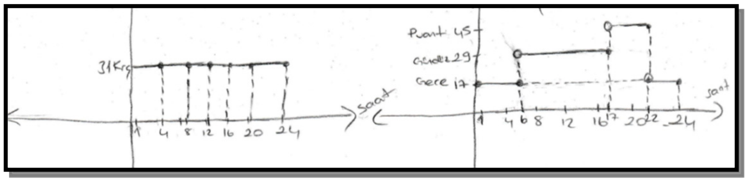


Figure 1. Graphs Drawn by the Students Together

In *the second part of this second problem* in the research form, a question in the form of "*What are the differences between these two graphs you have drawn?*" has been put forward by the observer. The answer that is expected by the researchers was in the form of; “there is no disconnection in one, but the other has” as was the case for the previous problem. However, students answered as “one is constant, but the other is not,” and this was not far distant from the expected answer. So, it is thought that what they implied by constant in dialogs in this section may be continuity.

In *the third part of the second problem* in the research form, students were addressed a question in the form of "*Please show the applied tariff as a time-dependent function?*” and it was analyzed whether students could express these tariffs mathematically. In this context, students had difficulties in defining the graphs as functions, but they were able to write some functions with the guidance of the observer. This shows that the students had difficulties in recognizing and building-with the concept of function. It is also noteworthy that students took the range of f(x) between 1 and 24 in which the function was defined despite the observer’s warnings. The expressions of the functions written for these graphs are presented below.

Figure 2. Expressions of Function Written by Zehra and Simge Together

In this section, students could not write the domain of the function, as seen. Moreover, although the observer explicitly warned the students in their answers, they designated the range for f(x) between 1 and 24 for the first graph. It is understood from this situation that the students intended to determine the closed-range of the independent variable x, not the f(x). When looking at the graphs, they had drawn and their statements in this section, it is seen that they know that the variable whose range they have determined is x, but they were not able to express this correctly when writing. However, it can be seen that Simge has determined ranges for the x in statements she wrote for the second function, but after she wrote the ranges down, she did not go back and correct by creating awareness.

In *the fourth part of the second problem* in the research form, a question with the form of "*Approach at both the graphs from right-hand and left-hand at 6 o’clock. Is there a relationship between the value of the function at that point and these limits?*" was addressed. The expressions of students in this section show that they recognized convergence and limit knowledge and built-with these. On the other hand, when Simge approached 6 o’clock from the right-hand, her answer was 17 instead of 29. This suggests that Simge did not read the graph correctly or she may have established an expectation that the limit was 17. After the guidance by the observer, Zehra expressed that there were two points in that section. Then, the students discussed the validity of these two points and decided that 17 must be the valid answer. This makes one think that the students have a misconception on that the limit value at point 6 is the value of the function at that point. These students, however, tried to approach the point 6 through the graph from right- and left-hand after the observer’s warning.

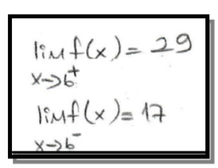


Figure 3. Limit Values Written by the Students Together

It was observed that the students have the knowledge of *reading a graph* and *approaching to a point*, but they could not have *built-with* these at a sufficient level. That is because the observer found it necessary to guide them in many instances. It was Simge, who stated that 29 is obtained when the point 6 is approached from the right-hand. In the answer for approaching from the left-hand, Simge expressed 45 as the answer and Zehra corrected her. Finally, the observer asked the students about the value of the function at that point and thus tried to ensure that the students would be able to reach the formal definition of continuity. In this section, it was observed that students could express the values of the function at that point correctly.

In *the final part of the second problem* in the research form, a question in the form of “*Can you produce a new* *concept for this relationship?*" was addressed and the transcript of the discussion is given below.

S181: Different results are coming up consistently.

Z182: You know, it appears in problem 1, when we approach from right-hand and left-hand, it is always the same. But when we approached it from the right-hand, it was 29 and when we approached it from the left-hand, it was 17. You know, there is no limit in problem 1, but there is here.

A183: All right, what do you think about the values of the function at that point?

S184: Actually, we have said that it was constant for the first one. The same value comes up all the time when we approach from the right-hand and the left-hand. But the second one is not constant and different results come up when we approach from the right-hand and the left-hand.

A185: Value of the function at that point…

S186: When we write down 6 to the function as the value of the function at that point…

A187: Can you produce a concept that is related to… ?

Z188: Will it be valid for these two graphs? Is there one thing in common?

S189: It states how we could explain the concept. What is it when we write down 6 to the f(x) function?

Z190: Miss, there is no limit.

A191: Ok, you said there is no limit but does not the value of the function at that point show any differences?

S192: It shows some differences...

Z193: You know, we have seen closeness-openness for this one, but it is all closed for this one and for example, openness exists for that one.

A194: All right, can you produce a new concept? What sort of concept could this be?

S195: If there are different values, openness, and closeness…

Z196: We have to think only for the functions, haven't we? (They stare at each other…). When we show the coordinates, you know, it is different for its graph, for example…

S198: The concept we are going to produce, is it related to the difference?

A199: Yes …

Z200: See, it is the closeness-openness feature.

Z201: There is both the closeness-openness feature and limit values of functions are different. Limit exists in one of them but doesn't exist in the other.

The expression of Zehra in Z182 here shows that this student built-with the knowledge of *limit should exist at the analyzed point* which is one of the three conditions required for the continuity. The statement of Simge in S184 in the form of "one of them is constant and the other is not" shows that this student refers to the disconnection on the second graph. However, an expression in the form of "*the existing limit should be equal to the value of the function at that point*" which is the formal definition of continuity, was not encountered in these dialogs. This shows that students have started to *construct* the knowledge of continuity, but they have not completed this process of knowledge abstraction.

4. Discussion

The students used the word "disconnection" in their responses to the research problems that are related to daily life. Here there is an approach to discontinuity. In the first problem in the research form, the students *recognized* the lights and switches in the circuit and *built-with* these the connections by stating the disconnection and interruption. This showed that students began to generate continuity knowledge and were ready to move to the second problem in the research form.

The abstraction process was partially completed in the second research problem with the statement of "*limit exists in one of them but doesn't exist in the other* [Z201]." In the second research problem, the students, who knew the concept of limit, also expressed the limits of the function at certain points correctly by taking limits from right-hand and left-hand. However, students were not able to express the definition of continuity mathematically.

As is known, there are three conditions for continuity. These are; the function should be defined at the point examined, the function should have the limit value at the point examined, and the value of the function and the limit value should be same at the point examined. In the interview, the students were able to fulfill the requirement that the limit value should be present at the point examined in relation to continuity. This is consistent with the result of the research conducted by Aydın and Kutluca (2010) which concludes that the students who intuitively feel continuity explained the concept of continuity by taking only right-hand and left-hand limit and do not look at whether the function is defined at the point examined. In addition, in the research by Baştürk and Dönmez (2011), students stated that if a function has a limit at a point, it is defined and continuous at that point. The result obtained in this study also coincides with this statement. This situation also supports the result of the research carried out by Bridgers (2007), which high school students were not able to correlate the concept of limit with continuity. This shows that these twelfth-grade students participating in the research have started to *construct* the knowledge of continuity but were not able to complete the process of knowledge abstraction. Nonetheless, although the students participating in this study could not complete the process, the desired outcomes could be reached by taking other subjects in mathematics into account. Although the students were successful in the limit concept, they could not establish the connection between limit, function, and continuity in this study. This result shows that the idea that the concept of function should be learned before the concept of continuity must be added to Mastorides and Zachariades (2004)'s idea; they had stated that learning the limit concept is a requirement before learning the continuity concept. The results of the study by Amatangelo (2013) support this idea. In that study, students have misconceptions because they cannot relate the function and limit concepts with continuity in a correct way. Accordingly, this affects *recognizing* and *building-with* actions in the abstraction process of continuity.

The results obtained in this research show that the students were able to *recognize* and *build-with* the concepts of limit, right- and left- convergence, which are necessary in order to construct the continuity concept. Although it is observed that the students *recognized* the knowledge of graph reading, they were not able to *build-with* this prior knowledge sufficiently for the solutions of the research problems. In the studies by Aydın and Kutluca (2010) and Takači, Pešić, and Tatar (2006), students also entrusted their vision to determine the continuity of a function on the graph. The students in this study utilized their vision to decide whether the functions are constant or not. This indicates that the students in this research do not have sufficient knowledge about the concepts of function, constant function, and piecewise-defined function, which are important to *construct* the continuity concept; therefore, the students were insufficient to *build-with* the concept of function in solving the second research problem. Areaya and Sidelil (2012) emphasized that students' lack of ability to draw graphs has an effect on understanding the concept of continuity. The students in this study were successful in the graph drawing of research problems. Therefore, it is not possible to say that the success of the graph drawing has an effect on understanding the concept of continuity within the scope of this research. However, the students' expressions on graphs of the functions (constant and not constant) indicated that they interpreted the jumps in the graph (Figure 1). So, this coincides with the *hole* interpretation of the students in studies by Areaya and Sidelil (2012) and Zvichapera (2016). Furthermore, the fact that the limit value at a point was perceived as the value of the function at this point suggests that the students have a misconception about this issue. So, these conceptions cause *constructing* action in the knowledge abstraction process not to be complete.

In this research, it was seen that the students from an Anatolian technical high school performed the abstraction process partially. Besides, as the knowledge could not be entirely generated, the *consolidation* action could not be implemented, and no data or results could be obtained. In order to complete the abstraction process regarding the continuity knowledge, it is appropriate to determine and eliminate the deficiencies and misconceptions in function, limit, and graph-reading and to study new research problems. The misconceptions have a considerable effect on the process of knowledge abstraction as seen from the study because the prior and existing knowledge should be active during the process. To make sure that the abstraction process will be achieved successfully, the abstraction process for the concept of function and limit need be examined for lower grade levels. This would provide sufficient prior knowledge for the abstraction of the continuity concept.

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