Reliable and Sophisticated Advanced Stressed Crises Compound Options Pricing Models

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Abstract

Compound Options are Exotic Derivatives Instruments that can be used to minimize the degree of investment risk, in hedging and speculative strategies which makes them Cheaper than the plain vanilla options and highly leveraged.From the traditional Robert Geske (1979) and Rubistein (1991) methods of compound options valuation (which simply assumed Normality) to Monte Carlo simulations method of valuation (which transform n – simulations of uniform variables into Normal Variables) were all underestimate (overestimate) the compound options prices because of the simple Normality Assumptions that governed the formulation of the models or process.Above all they lag the incorporation of Fat – Tailed Effects of returns on compound options underlying assets' probability distributions to enable them capture the popular Black Swan Events as propagated by Nassim N. Taleb and other professional risk management associations like PRMIA.However, in this paper, the Author attempted to incorporate fat – tailed effects into the models using Jameel's Contractional and Expansional Stress Methods so as to enable them precisely traces the trajectories of the Past and Future Black Swan events to avoid reoccurrences of future economic and financial crises related to exotic options more often otherwise makes the existing models more reliable, robust, sophisticated and holistic. Finally, the proposed Jameel's Advanced Stressed Exotic Options Pricing Models are expected to dramatically increase the markets Confidence and drastically decreases the markets Risks.

Keywords: *Strike; Exotic; Fat –tailed; Call; Put; Jameel*

1. Introduction

The Compound Option gives the holder the right to buy (in case of Call) or sell (in case of Put) the underlying Option. Compound options are options on options. The underlying asset is itself an Option.Compound Option has two strike prices (X_1: Compound and X_2: underlying) and two expiration dates (T_1: Compound and T_2: underlying with $T_1 < T_2$). At the first exercise date T_1 you must decide whether it is worth exercising the first option (depending on the strike price X_1 and the current asset price S). If so, you get a further option with strike price X_2 and maturity T_2.There are four basic types of compound options: **Call on Call (CoC), Call on Put (CoP) or caput option, Put on Put (PoP) and Put on Call (PoC).** The payoff of a compound option involves the value of another option.There are four possible payoffs: Call on call: $Max\{C(S,T_1) - X_1, 0\}$, Call on put: $Max\{P(S,T_1) - X_1, 0\}$, Put on call: $Max\{X_1 - C(S,T_1), 0\}$ and Put on put: $Max\{X_1 - P(S,T_1), 0\}$.

Robert Geske (1979) and Rubistein (1991) were the first to find the solution and implement the closed form formulae on the basis of the Black – Scholes – Merton Model (1973) to price European – type of compound options. Other research has been conducted by Thomassen and Van Wouwe (2002).

The overreliance of **NORMALITY ASSUMPTIONS** in the valuation process of Exotic Options (Compound

Options) has seriously and continuously threatens the resilient of economic and financial stability reference to the Exotic Options Pricing.Tremendously,the **Small Probabilities Margin** has significant **IMPACT** in our Exotic (Compound) Options Prices as propagated by Nassim N. Taleb and other world economic and financial leading actors.The Robert Geske (1979) and Rubistein (1991) method of valuing compound options assumed normality and constant volatility, so they out rightly underestimates the prices while the Monte Carlo simulations transform n – simulations of uniform variables into normal variables, however, these seriously **NEGLECTED** the potential effects of Small Probabilities Margin or Low – Probability, High – Impact events occurs at the fourth quadrant . These are of course thrown the risk of underestimating (overestimating) compound options prices to the entire world economic and financial systems.

In view of the foregoing, we need to **EXPLICITLY STRESS** all our Exotic Options Pricing Models by incorporating fat – tailed effects into them to enable them precisely capture potential Black Swans.

In this paper, the Author consider Robert Geske (1979) and Rubistein (1991) compound options pricing models and incorporates fat – tailed effects into them using Jameel'sContractional and Expansional Stress Methods.

2. Material and Method

2.1 Robert Geske (1979) – Rubistein (1991) Assumptions and Monte Carlo Simulations Method of Valuing Compound Options

2.1.1 Jameel's Observations

- Robert Geske (1979) and Rubinstein (1991) assumed NORMALITY and CONSTANT VOLATILITY because of this, the price obtained with the models are generally UNDERESTIMATED.

- The Monte Carlo Simulations Model implements n – Simulations of uniform variables which then transforms into NORMAL variables.

- The FAT – TAILED EFFECTS on the RETURNS of the underlying probability distributions of $S, X_1 \text{ and } X_2$ are NOT considered in both Robert Geske (1979), Rubistein (1991) and Monte Carlo Simulations Method.

- After the Stock market Crash of 1987, the concepts of VOLITILITY SMILE and SKEW came into account. That is fat – tailed effects in the underlying assets distributions. This concept critically analyses the presence of SKEW and HEAVY KUTOSIS in the options strikes prices over different volatilities. However, the existing models do not take into account the incorporation of volatility smile and skew.

- Furthermore, there is concrete evidence that the empirical returns distribution DOES NOT appear to be GAUSSIAN.

- Hence, there are great tendencies of the Compound (Exotic) Options Models to Underestimate (Overestimate) the prices especially at the times of economic recoveries or recessions.

That is why Jameel'sContractional and ExpansionalStress Methods addresses the above Observations as follows.

2.2 Method

The methodology adopted in this research work is to use Jameel'sContractional and Expansional stress methods to **SEVERELY** stress the traditional **Compound (Exotic) Options Pricing Models such as Robert Geske (1979), its Greeks, and Rubistein (1991)** using (a) Geometric Return of the Arithmetic Means of the U.S. macroeconomic indicators plus research company underlying stock returns μ_S and the Geometric volatility of the volatilities of the U.S. macroeconomic indicators plus research company underlying stock returns σ_S as well as the Best fitted fat – tailed probability distribution of the research company underlying stock returns $f_3\left(s, \mu_S, \sigma_S, \xi\right)$ (b) Geometric Return of the Arithmetic Means of the U.S. macroeconomic indicators plus research company simple underlying strike returns μ_{X_2} and the Geometric volatility of the volatilities of the U.S. macroeconomic indicators plus research company simple underlying strike returns σ_{X_2} (c) Geometric Return of the Arithmetic Means of the U.S.

macroeconomic indicators plus research company compound underlying strike returns μ_{X_1} and the Geometric volatility of the volatilities of the U.S. macroeconomic indicators plus research company compound underlying strike returns σ_{X_1} as well as the Best fitted fat – tailed probability distribution of the research companysimple and compound underlying strikesreturns $f_2(x_2, \mu, \sigma, \pi)$ and $f_1(x_1, \mu, \sigma, \pi)$ (which are the **BEST FITTED**Fat – Tailed Probability Distributions of the Compound Options' Two Strikes obtained using Jameel's Criterion) respectively (c) Geometric Return of the Arithmetic Means of the U.S. macroeconomic indicators plus research company underlying stock returns and underlying two strikesreturns $\mu_{S X_1 X_2}$ and the Geometric volatility of the volatilities of the U.S. macroeconomic indicators plus research company underlying stock returns and underlying two strikesreturns $\sigma_{S X_1 X_2}$ given that X_1: is the underlying compound option Strike Price and X_2: is the simple underlying Strike Price as shown below:
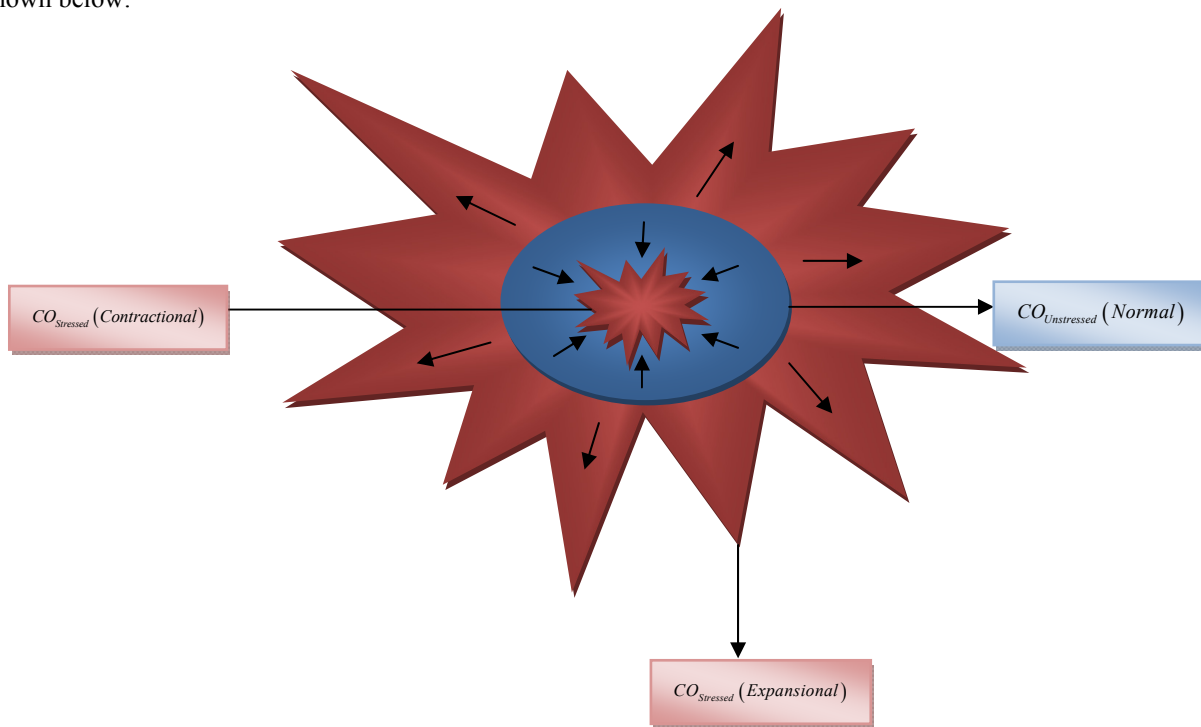

Figure 1. Jameel's Contractional – Expansional Stress Diagram

2.2.1 Proposed Jameel's CALL on CALL Stressed Option Models

Class 1:

$$CC_{Stressed} = Se^{-qT_2}\left[N_2\left(\mu_{X_1}\mu_{X_2}\mu_S\left(a_1, b_1; \sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right]$$
$$- X_2 e^{-rT_2}\left[N_2\left(\mu_{X_1}\mu_{X_2}\mu_S\left(a_2, b_2; \sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right]$$
$$- X_1 e^{-rT_1}\left[N\left(\mu_{X_1}\mu_{X_2}\mu_S\left(a_2\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right]$$

Class 2:

$$CC_{Stressed} = Se^{-qT_2}\left[N_2\left(\mu_{X_1X_2S}\left(a_1, b_1; \sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1X_2S}f_1(.) \pm \sigma_{X_1X_2S}f_2(.) \pm \sigma_{X_1X_2S}f_3(.)\right]$$
$$- X_2e^{-rT_2}\left[N_2\left(\mu_{X_1X_2S}\left(a_2, b_2; \sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1X_2S}f_1(.) \pm \sigma_{X_1X_2S}f_2(.) \pm \sigma_{X_1X_2S}f_3(.)\right]$$
$$- X_1e^{-rT_1}\left[N\left(\mu_{X_1X_2S}\left(a_2\right)\right) \pm \sigma_{X_1X_2S}f_1(.) \pm \sigma_{X_1X_2S}f_2(.) \pm \sigma_{X_1X_2S}f_3(.)\right]$$

Where,

$$CC = Se^{-qT_2}N_2\left(a_1, b_1; \sqrt{T_1/T_2}\right)$$
$$- X_2e^{-rT_2}N_2\left(a_2, b_2; \sqrt{T_1/T_2}\right)$$
$$- X_1e^{-rT_1}N\left(a_2\right)$$

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $CC_{Stressed}$: positive, negative, high, low, close and so on).CLASS 1 and 2 are **OPTIMUM** at:

$$CC_{Stressed} = Se^{-qT_2}\left[N_2\left(a_1, b_1; \sqrt{T_1/T_2}\right) \pm \sigma_{X_1}f_1(.) \pm \sigma_{X_2}f_2(.) \pm \sigma_S f_3(.)\right]$$
$$- X_2e^{-rT_2}\left[N_2\left(a_2, b_2; \sqrt{T_1/T_2}\right) \pm \sigma_{X_1}f_1(.) \pm \sigma_{X_2}f_2(.) \pm \sigma_S f_3(.)\right]$$
$$- X_1e^{-rT_1}\left[N\left(a_2\right) \pm \sigma_{X_1}f_1(.) \pm \sigma_{X_2}f_2(.) \pm \sigma_S f_3(.)\right]$$

For some σ_{X_i}'s $= 1$ $(i = 1, 2)$ with (i) All $\mu_{X_1} = \mu_{X_2} = \mu_S = 1$ (ii) Some μ_{X_i}'s $= 1$ $(i = 1, 2)$

And/or

$$CC_{Stressed} = Se^{-qT_2}\left[N_2\left(a_1, b_1; \sqrt{T_1/T_2}\right) \pm \sigma_{X_1X_2S}f_1(.) \pm \sigma_{X_1X_2S}f_2(.) \pm \sigma_{X_1X_2S}f_3(.)\right]$$
$$- X_2e^{-rT_2}\left[N_2\left(a_2, b_2; \sqrt{T_1/T_2}\right) \pm \sigma_{X_1X_2S}f_1(.) \pm \sigma_{X_1X_2S}f_2(.) \pm \sigma_{X_1X_2S}f_3(.)\right]$$
$$- X_1e^{-rT_1}\left[N\left(a_2\right) \pm \sigma_{X_1X_2S}f_1(.) \pm \sigma_{X_1X_2S}f_2(.) \pm \sigma_{X_1X_2S}f_3(.)\right]$$

For some $\sigma_{X_iX_iS}$'s $= 1$ $(i = 1, 2)$ with (i) All $\mu_{X_1X_2S} = 1$ (ii) Some $\mu_{X_1X_2S}$'s $= 1$ $(i = 1, 2)$

2.2.2 Proposed Jameel's CALL on PUT Stressed Option Models
Class 1:

$$CP_{Stressed} = X_2e^{-rT_2}\left[N_2\left(\mu_{X_1}\mu_{X_2}\mu_S\left(-a_2, -b_2; \sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1}f_1(.) \pm \sigma_{X_2}f_2(.) \pm \sigma_S f_3(.)\right]$$
$$- Se^{-qT_2}\left[N_2\left(\mu_{X_1}\mu_{X_2}\mu_S\left(-a_1, -b_1; \sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1}f_1(.) \pm \sigma_{X_2}f_2(.) \pm \sigma_S f_3(.)\right]$$
$$- X_1e^{-rT_1}\left[N\left(\mu_{X_1}\mu_{X_2}\mu_S\left(-a_2\right)\right) \pm \sigma_{X_1}f_1(.) \pm \sigma_{X_2}f_2(.) \pm \sigma_S f_3(.)\right]$$

Class 2:

$$CP_{Stressed} = X_2e^{-rT_2}\left[N_2\left(\mu_{X_1X_2S}\left(-a_2, -b_2; \sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1X_2S}f_1(.) \pm \sigma_{X_1X_2S}f_2(.) \pm \sigma_{X_1X_2S}f_3(.)\right]$$
$$- Se^{-qT_2}\left[N_2\left(\mu_{X_1X_2S}\left(-a_1, -b_1; \sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1X_2S}f_1(.) \pm \sigma_{X_1X_2S}f_2(.) \pm \sigma_{X_1X_2S}f_3(.)\right]$$
$$- X_1e^{-rT_1}\left[N\left(\mu_{X_1X_2S}\left(-a_2\right)\right) \pm \sigma_{X_1X_2S}f_1(.) \pm \sigma_{X_1X_2S}f_2(.) \pm \sigma_{X_1X_2S}f_3(.)\right]$$

Where,

$$CP_{Stressed} = X_2e^{-rT_2}N_2\left(-a_2, -b_2; \sqrt{T_1/T_2}\right)$$
$$- Se^{-qT_2}N_2\left(-a_1, -b_1; \sqrt{T_1/T_2}\right)$$
$$- X_1e^{-rT_1}N\left(-a_2\right)$$

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $CP_{Stressed}$: positive, negative, high, low, close and so on).CLASS 1 and 2 are **OPTIMUM** at:

$$CP_{Stressed} = X_2 e^{-rT_2} \left[N_2 \left(-a_2, -b_2; \sqrt{T_1/T_2} \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$
$$- S e^{-qT_2} \left[N_2 \left(-a_1, -b_1; \sqrt{T_1/T_2} \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$
$$- X_1 e^{-rT_1} \left[N \left(-a_2 \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$

For some σ_{X_i} 's $= 1 \ (i = 1, 2)$ with (i) All $\mu_{X_1} = \mu_{X_2} = \mu_S = 1$ (ii) Some μ_{X_i} 's $= 1 \ (i = 1, 2)$

And/or

$$CP_{Stressed} = X_2 e^{-rT_2} \left[N_2 \left(-a_2, -b_2; \sqrt{T_1/T_2} \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$
$$- S e^{-qT_2} \left[N_2 \left(-a_1, -b_1; \sqrt{T_1/T_2} \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$
$$- X_1 e^{-rT_1} \left[N \left(-a_2 \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$

For some $\sigma_{X_i X_i S}$'s $= 1 \ (i = 1, 2)$ with (i) All $\mu_{X_1 X_2 S} = 1$ (ii) Some $\mu_{X_1 X_2 S}$'s $= 1 \ (i = 1, 2)$

2.2.3 Proposed Jameel's PUT on CALL Stressed Option Models

Class 1:

$$PC_{Stressed} = X_2 e^{-rT_2} \left[N_2 \left(\mu_{X_1} \mu_{X_2} \mu_S \left(-a_2, b_2; -\sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$
$$- S e^{-qT_2} \left[N_2 \left(\mu_{X_1} \mu_{X_2} \mu_S \left(-a_1, -b_1; -\sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$
$$+ X_1 e^{-rT_1} \left[N \left(\mu_{X_1} \mu_{X_2} \mu_S \left(-a_2 \right) \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$

Class 2:

$$PC_{Stressed} = X_2 e^{-rT_2} \left[N_2 \left(\mu_{X_1 X_2 S} \left(-a_2, b_2; -\sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$
$$- S e^{-qT_2} \left[N_2 \left(\mu_{X_1 X_2 S} \left(-a_1, -b_1; -\sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$
$$+ X_1 e^{-rT_1} \left[N \left(\mu_{X_1 X_2 S} \left(-a_2 \right) \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$

Where,

$$PC = X_2 e^{-rT_2} \left[N_2 \left(-a_2, b_2; -\sqrt{T_1/T_2} \right) \right]$$
$$- S e^{-qT_2} \left[N_2 \left(-a_1, -b_1; -\sqrt{T_1/T_2} \right) \right]$$
$$+ X_1 e^{-rT_1} \left[N \left(-a_2 \right) \right]$$

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $PC_{Stressed}$: positive, negative, high, low, close and so on).CLASS 1 and 2 are **OPTIMUM** at:

$$PC_{Stressed} = X_2 e^{-rT_2} \left[N_2 \left(-a_2, b_2; -\sqrt{T_1/T_2} \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$
$$- Se^{-qT_2} \left[N_2 \left(-a_1, -b_1; -\sqrt{T_1/T_2} \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$
$$+ X_1 e^{-rT_1} \left[N \left(-a_2 \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$

For some σ_{X_i}'s $= 1 \, (i = 1,2)$ with (i) All $\mu_{X_1} = \mu_{X_2} = \mu_S = 1$ (ii) Some μ_{X_i}'s $= 1 \, (i = 1,2)$

And/or

$$PC_{Stressed} = X_2 e^{-rT_2} \left[N_2 \left(-a_2, b_2; -\sqrt{T_1/T_2} \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$
$$- Se^{-qT_2} \left[N_2 \left(-a_1, -b_1; -\sqrt{T_1/T_2} \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$
$$+ X_1 e^{-rT_1} \left[N \left(-a_2 \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$

For some $\sigma_{X_i X_i S}$'s $= 1 \, (i = 1,2)$ with (i) All $\mu_{X_1 X_2 S} = 1$ (ii) Some $\mu_{X_1 X_2 S}$'s $= 1 \, (i = 1,2)$

2.2.4 Proposed Jameel's PUT on PUT Stressed Option Models

Class 1:

$$PP_{Stressed} = Se^{-qT_2} \left[N_2 \left(\mu_{X_1} \mu_{X_2} \mu_S \left(a_1, -b_1; -\sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$
$$- X_2 e^{-rT_2} \left[N_2 \left(\mu_{X_1} \mu_{X_2} \mu_S \left(a_2, -b_2; -\sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$
$$+ X_1 e^{-rT_1} \left[N \left(\mu_{X_1} \mu_{X_2} \mu_S \left(a_2 \right) \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$

Class 2:

$$PP_{Stressed} = Se^{-qT_2} \left[N_2 \left(\mu_{X_1 X_2 S} \left(a_1, -b_1; -\sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$
$$- X_2 e^{-rT_2} \left[N_2 \left(\mu_{X_1 X_2 S} \left(a_2, -b_2; -\sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$
$$+ X_1 e^{-rT_1} \left[N \left(\mu_{X_1 X_2 S} \left(a_2 \right) \right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.) \right]$$

Where,

$$PP = Se^{-qT_2} \left[N_2 \left(a_1, -b_1; -\sqrt{T_1/T_2} \right) \right]$$
$$- X_2 e^{-rT_2} \left[N_2 \left(a_2, -b_2; -\sqrt{T_1/T_2} \right) \right]$$
$$+ X_1 e^{-rT_1} \left[N \left(a_2 \right) \right]$$

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $PP_{Stressed}$: positive, negative, high, low, close and so on).CLASS 1 and 2 are **OPTIMUM** at:

$$PP_{Stressed} = Se^{-qT_2} \left[N_2 \left(a_1, -b_1; -\sqrt{T_1/T_2} \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$
$$- X_2 e^{-rT_2} \left[N_2 \left(a_2, -b_2; -\sqrt{T_1/T_2} \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$
$$+ X_1 e^{-rT_1} \left[N \left(a_2 \right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_s f_3(.) \right]$$

For some σ_{X_i}'s $= 1 \, (i = 1,2)$ with (i) All $\mu_{X_1} = \mu_{X_2} = \mu_S = 1$ (ii) Some μ_{X_i}'s $= 1 \, (i = 1,2)$

And/or

$$PP_{Stressed} = Se^{-qT_2}\left[N_2\left(a_1, -b_1; -\sqrt{T_1/T_2}\right) \pm \sigma_{X_1X_2S} f_1(.) \pm \sigma_{X_1X_2S} f_2(.) \pm \sigma_{X_1X_2S} f_3(.)\right]$$
$$- X_2 e^{-rT_2}\left[N_2\left(a_2, -b_2; -\sqrt{T_1/T_2}\right) \pm \sigma_{X_1X_2S} f_1(.) \pm \sigma_{X_1X_2S} f_2(.) \pm \sigma_{X_1X_2S} f_3(.)\right]$$
$$+ X_1 e^{-rT_1}\left[N\left(a_2\right) \pm \sigma_{X_1X_2S} f_1(.) \pm \sigma_{X_1X_2S} f_2(.) \pm \sigma_{X_1X_2S} f_3(.)\right]$$

For some $\sigma_{X_iX_iS}$'s $= 1$ $(i = 1,2)$ with (i) All $\mu_{X_1X_2S} = 1$ (ii) Some $\mu_{X_1X_2S}$'s $= 1$ $(i = 1,2)$

Where, $N_2\left(a, b; \sqrt{T_1/T_2}\right)$ is the cumulative distribution function of the BIVARIATE Normal Random Variable

with Time $\sqrt{T_1/T_2}$, that is

$$N(a_2) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{a_2} e^{-\frac{(a_2-\mu)^2}{2\sigma^2}} da_2 , \quad N_2\left(a_2, b_2; \sqrt{T_1/T_2}\right) = \frac{1}{2\pi\sqrt{1-\left(\sqrt{T_1/T_2}\right)^2}}\int_{-\infty}^{a}\int_{-\infty}^{b} e^{-\frac{1}{2\left(1-\left(\sqrt{T_1/T_2}\right)^2\right)}\left(a^2 - 2\left(\sqrt{T_1/T_2}\right)ab + b^2\right)} da\, db$$

$$a_1 = \frac{\log(S/X_2) + (r + \sigma^2/2)T_2}{\sigma\sqrt{T_2}} , \quad b_1 = \frac{\log(S/X_1) + (r + \sigma^2/2)T_1}{\sigma\sqrt{T_1}} , \quad a_2 = a_1 - \sigma\sqrt{t_n} \text{ and } b_2 = b_1 - \sigma\sqrt{t_1} .$$

2.3 Proposed Jameel's Stressed Greeks

2.3.1 Proposed Jameel's Stressed DELTA Models

Class 1:

$$\left(\frac{\partial V}{\partial S}\right)_{Stressed} = \phi\omega r_f Se^{-r_f T_2}\left[N_2\left(-\mu_{X_1}\mu_{X_2}\mu_S \phi w\left(X_2 - \sigma\sqrt{T_1}, \phi d_+; \omega\sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right]$$
$$- \phi\omega r_d X_1 e^{-r_d T_2}\left[N_2\left(-\mu_{X_1}\mu_{X_2}\mu_S \phi w\left(X_2, \phi d_-; \omega\sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right]$$
$$- \omega r_d X_1 e^{-r_d T_1}\left[N\left(-\mu_{X_1}\mu_{X_2}\mu_S \phi w X_2\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right]$$
$$- \frac{\sigma}{2} Se^{-r_f T_2}\left[\frac{1}{\sqrt{t}} n\left(X_2 - \sigma\sqrt{T_1}\right)\left(N\left(\mu_{X_1}\mu_{X_2}\mu_S \phi d_+^{T_2}\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right)\right]$$
$$+ \frac{\omega}{\sqrt{T_2}} n(d_+)\left[N\left(\mu_{X_1}\mu_{X_2}\mu_S\left(-\phi\omega\frac{X_2\sqrt{T_2}+\sqrt{T_1}d_-}{\sqrt{\tau}}\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right]$$

Class 2:

$$\left(\frac{\partial V}{\partial S}\right)_{Stressed} = \phi\omega r_f Se^{-r_f T_2}\left[N_2\left(-\mu_{X_1X_2S} \phi w\left(X_2 - \sigma\sqrt{T_1}, \phi d_+; \omega\sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1X_2S} f_1(.) \pm \sigma_{X_1X_2S} f_2(.) \pm \sigma_{X_1X_2S} f_3(.)\right]$$
$$- \phi\omega r_d X_1 e^{-r_d T_2}\left[N_2\left(-\mu_{X_1X_2S} \phi w\left(X_2, \phi d_-; \omega\sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1X_2S} f_1(.) \pm \sigma_{X_1X_2S} f_2(.) \pm \sigma_{X_1X_2S} f_3(.)\right]$$
$$- \omega r_d X_1 e^{-r_d T_1}\left[N\left(-\mu_{X_1X_2S} \phi w X_2\right) \pm \sigma_{X_1X_2S} f_1(.) \pm \sigma_{X_1X_2S} f_2(.) \pm \sigma_{X_1X_2S} f_3(.)\right]$$
$$- \frac{\sigma}{2} Se^{-r_f T_2}\left[\frac{1}{\sqrt{t}} n\left(X_2 - \sigma\sqrt{T_1}\right)\left(N\left(\mu_{X_1X_2S} \phi d_+^{T_2}\right) \pm \sigma_{X_1X_2S} f_1(.) \pm \sigma_{X_1X_2S} f_2(.) \pm \sigma_{X_1X_2S} f_3(.)\right)\right]$$
$$+ \frac{\omega}{\sqrt{T_2}} n(d_+)\left[N\left(\mu_{X_1X_2S}\left(-\phi\omega\frac{X_2\sqrt{T_2}+\sqrt{T_1}d_-}{\sqrt{\tau}}\right)\right) \pm \sigma_{X_1X_2S} f_1(.) \pm \sigma_{X_1X_2S} f_2(.) \pm \sigma_{X_1X_2S} f_3(.)\right]$$

Where,

$$
\begin{aligned}
\frac{\partial V}{\partial S} = {} & \phi \omega r_f S e^{-r_f T_2} N_2 \left(-\phi w \left(X_2 - \sigma \sqrt{T_1}, \phi d_+ ; \omega \sqrt{T_1/T_2} \right) \right) \\
& - \phi \omega r_d X_1 e^{-r_d T_2} N_2 \left(-\phi w \left(X_2, \phi d_- ; \omega \sqrt{T_1/T_2} \right) \right) \\
& - \omega r_d X_1 e^{-r_d T_1} N \left(-\phi w X_2 \right) \\
& - \frac{\sigma}{2} S e^{-r_f T_2} \left[\frac{1}{\sqrt{t}} n \left(X_2 - \sigma \sqrt{T_1} \right) N \left(\phi d_+^{T_2} \right) \right] \\
& + \frac{\omega}{\sqrt{T_2}} n \left(d_+ \right) N \left(-\phi \omega \frac{X_2 \sqrt{T_2} + \sqrt{T_1} d_-}{\sqrt{\tau}} \right)
\end{aligned}
$$

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $\left(\dfrac{\partial V}{\partial S} \right)_{Stressed}$: positive, negative, high, low, close and so on).CLASS 1 and 2 are **OPTIMUM** at:

$$
\begin{aligned}
\left(\frac{\partial V}{\partial S} \right)_{Stressed} = {} & \phi \omega r_f S e^{-r_f T_2} \left[N_2 \left(-\phi w \left(X_2 - \sigma \sqrt{T_1}, \phi d_+ ; \omega \sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1} f_1 (.) \pm \sigma_{X_2} f_2 (.) \pm \sigma_s f_3 (.) \right] \\
& - \phi \omega r_d X_1 e^{-r_d T_2} \left[N_2 \left(-\phi w \left(X_2, \phi d_- ; \omega \sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1} f_1 (.) \pm \sigma_{X_2} f_2 (.) \pm \sigma_s f_3 (.) \right] \\
& - \omega r_d X_1 e^{-r_d T_1} \left[N \left(-\phi w X_2 \right) \pm \sigma_{X_1} f_1 (.) \pm \sigma_{X_2} f_2 (.) \pm \sigma_s f_3 (.) \right] \\
& - \frac{\sigma}{2} S e^{-r_f T_2} \left[\frac{1}{\sqrt{t}} n \left(X_2 - \sigma \sqrt{T_1} \right) \left(N \left(\phi d_+^{T_2} \right) \pm \sigma_{X_1} f_1 (.) \pm \sigma_{X_2} f_2 (.) \pm \sigma_s f_3 (.) \right) \right] \\
& + \frac{\omega}{\sqrt{T_2}} n \left(d_+ \right) \left[N \left(\left(-\phi \omega \frac{X_2 \sqrt{T_2} + \sqrt{T_1} d_-}{\sqrt{\tau}} \right) \right) \pm \sigma_{X_1} f_1 (.) \pm \sigma_{X_2} f_2 (.) \pm \sigma_s f_3 (.) \right]
\end{aligned}
$$

For some σ_{X_i}'s $=1 \, (i=1,2)$ with (i) All $\mu_{X_1} = \mu_{X_2} = \mu_S = 1$ (ii) Some μ_{X_i}'s $=1 \, (i=1,2)$

And/or

$$
\begin{aligned}
\left(\frac{\partial V}{\partial S} \right)_{Stressed} = {} & \phi \omega r_f S e^{-r_f T_2} \left[N_2 \left(-\phi w \left(X_2 - \sigma \sqrt{T_1}, \phi d_+ ; \omega \sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1 X_2 S} f_1 (.) \pm \sigma_{X_1 X_2 S} f_2 (.) \pm \sigma_{X_1 X_2 S} f_3 (.) \right] \\
& - \phi \omega r_d X_1 e^{-r_d T_2} \left[N_2 \left(-\phi w \left(X_2, \phi d_- ; \omega \sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1 X_2 S} f_1 (.) \pm \sigma_{X_1 X_2 S} f_2 (.) \pm \sigma_{X_1 X_2 S} f_3 (.) \right] \\
& - \omega r_d X_1 e^{-r_d T_1} \left[N \left(-\phi w X_2 \right) \pm \sigma_{X_1 X_2 S} f_1 (.) \pm \sigma_{X_1 X_2 S} f_2 (.) \pm \sigma_{X_1 X_2 S} f_3 (.) \right] \\
& - \frac{\sigma}{2} S e^{-r_f T_2} \left[\frac{1}{\sqrt{t}} n \left(X_2 - \sigma \sqrt{T_1} \right) \left(N \left(\phi d_+^{T_2} \right) \pm \sigma_{X_1 X_2 S} f_1 (.) \pm \sigma_{X_1 X_2 S} f_2 (.) \pm \sigma_{X_1 X_2 S} f_3 (.) \right) \right] \\
& + \frac{\omega}{\sqrt{T_2}} n \left(d_+ \right) \left[N \left(\left(-\phi \omega \frac{X_2 \sqrt{T_2} + \sqrt{T_1} d_-}{\sqrt{\tau}} \right) \right) \pm \sigma_{X_1 X_2 S} f_1 (.) \pm \sigma_{X_1 X_2 S} f_2 (.) \pm \sigma_{X_1 X_2 S} f_3 (.) \right]
\end{aligned}
$$

For some $\sigma_{X_i X_i S}$'s $=1 \, (i=1,2)$ with (i) All $\mu_{X_1 X_2 S} = 1$ (ii) Some $\mu_{X_1 X_2 S}$'s $=1 \, (i=1,2)$

2.3.2 ProposedJameel's Stressed GAMMA Models

Class 1:

$$
\left(\frac{\partial^2 V}{\partial S^2} \right)_{Stressed} = \phi \omega e^{-r_f T_2} \frac{\partial}{\partial S} \left[N_2 \left(-\mu_{X_1} \mu_{X_2} \mu_S \phi w \left(X_2 - \sigma \sqrt{T_1}, \phi d_+ ; \omega \sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1} f_1 (.) \pm \sigma_{X_2} f_2 (.) \pm \sigma_S f_3 (.) \right]
$$

Class 2:

$$
\left(\frac{\partial^2 V}{\partial S^2} \right)_{Stressed} = \phi \omega e^{-r_f T_2} \frac{\partial}{\partial S} \left[N_2 \left(-\mu_{X_1 X_2 S} \phi w \left(X_2 - \sigma \sqrt{T_1}, \phi d_+ ; \omega \sqrt{T_1/T_2} \right) \right) \pm \sigma_{X_1 X_2 S} f_1 (.) \pm \sigma_{X_1 X_2 S} f_2 (.) \pm \sigma_{X_1 X_2 S} f_3 (.) \right]
$$

Where, $\dfrac{\partial^2 V}{\partial S^2} = \phi \omega e^{-r_f T_2} \dfrac{\partial}{\partial S}\left[N_2\left(-\phi w\left(X_2 - \sigma\sqrt{T_1}, \phi d_+; \omega\sqrt{T_1/T_2}\right)\right)\right]$. These are the general FORMS of the proposal,

however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to

achieve different values of $\left(\dfrac{\partial^2 V}{\partial S^2}\right)_{Stressed}$: positive, negative, high, low, close and so on).We can find the **OPTIMUM**

values of **GAMMA CLASS 1 and 2** in the same way as in the case of **DELTA CLASS 1 and 2** above.

2.3.3 Proposed Jameel's Stressed THETA Models

Class 1:

$$
\begin{aligned}
\left(\frac{\partial V}{\partial T}\right)_{Stressed} =\ & \phi\omega r_f S e^{-r_f T_2}\left[N_2\left(-\mu_{X_1}\mu_{X_2}\mu_S \phi w\left(X_2 - \sigma\sqrt{T_1}, \phi d_+; \omega\sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right] \\
& - \phi\omega r_d X_1 e^{-r_d T_2}\left[N_2\left(-\mu_{X_1}\mu_{X_2}\mu_S \phi w\left(X_2, \phi d_-; \omega\sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right] \\
& - \omega r_d X_1 e^{-r_d T_1}\left[N\left(-\mu_{X_1}\mu_{X_2}\mu_S \phi w X_2\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right] \\
& - \frac{\sigma}{2} S e^{-r_f T_2}\left[\begin{array}{l} \dfrac{1}{\sqrt{t}} n\left(X_2 - \sigma\sqrt{T_1}\right)\left(N\left(\mu_{X_1}\mu_{X_2}\mu_S \phi d_+^{\tau}\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right) \\ + \dfrac{\omega}{\sqrt{T_2}} n(d_+)\left(N\left(\mu_{X_1}\mu_{X_2}\mu_S\left(-\phi\omega\dfrac{X_2\sqrt{T_2} + \sqrt{T_1}d_-}{\sqrt{\tau}}\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right) \end{array}\right]
\end{aligned}
$$

Class 2:

$$
\begin{aligned}
\left(\frac{\partial V}{\partial T}\right)_{Stressed} =\ & \phi\omega r_f S e^{-r_f T_2}\left[N_2\left(-\mu_{X_1 X_2 S}\phi w\left(X_2 - \sigma\sqrt{T_1}, \phi d_+; \omega\sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.)\right] \\
& - \phi\omega r_d X_1 e^{-r_d T_2}\left[N_2\left(-\mu_{X_1 X_2 S}\phi w\left(X_2, \phi d_-; \omega\sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.)\right] \\
& - \omega r_d X_1 e^{-r_d T_1}\left[N\left(-\mu_{X_1 X_2 S}\phi w X_2\right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.)\right] \\
& - \frac{\sigma}{2} S e^{-r_f T_2}\left[\begin{array}{l} \dfrac{1}{\sqrt{t}} n\left(X_2 - \sigma\sqrt{T_1}\right)\left(N\left(\mu_{X_1 X_2 S}\phi d_+^{\tau}\right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.)\right) \\ + \dfrac{\omega}{\sqrt{T_2}} n(d_+)\left(N\left(\mu_{X_1 X_2 S}\left(-\phi\omega\dfrac{X_2\sqrt{T_2} + \sqrt{T_1}d_-}{\sqrt{\tau}}\right)\right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.)\right) \end{array}\right]
\end{aligned}
$$

Where,

$$
\begin{aligned}
\frac{\partial V}{\partial T} =\ & \phi\omega r_f S e^{-r_f T_2} N_2\left(-\phi w\left(X_2 - \sigma\sqrt{T_1}, \phi d_+; \omega\sqrt{T_1/T_2}\right)\right) \\
& - \phi\omega r_d X_1 e^{-r_d T_2} N_2\left(-\phi w\left(X_2, \phi d_-; \omega\sqrt{T_1/T_2}\right)\right) \\
& - \omega r_d X_1 e^{-r_d T_1} N\left(-\phi w X_2\right) \\
& - \frac{\sigma}{2} S e^{-r_f T_2}\left[\frac{1}{\sqrt{t}} n\left(X_2 - \sigma\sqrt{T_1}\right) N\left(\phi d_+^{\tau}\right) + \frac{\omega}{\sqrt{T_2}} n(d_+) N\left(-\phi\omega\frac{X_2\sqrt{T_2} + \sqrt{T_1}d_-}{\sqrt{\tau}}\right)\right]
\end{aligned}
$$

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible

Combination of TERMS and SIGNS (to achieve different values of $\left(\dfrac{\partial V}{\partial T}\right)_{Stressed}$: positive, negative, high, low, close

and so on).We can find the **OPTIMUM** values of **THETA CLASS 1 and 2** in the same way as in the case of **DELTA CLASS 1 and 2** above.

2.3.4 Proposed Jameel's Stressed RHO Models

Class 1:

$$
\left(\frac{\partial V}{\partial r_f}\right)_{Stressed} = -\phi\omega T_2 S e^{-r_f T_2}\left[N_2\left(-\mu_{X_1}\mu_{X_2}\mu_S \phi\omega\left(X_2 - \sigma\sqrt{T_1}, \phi d_+; \omega\sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right]
$$

Class 2:

$$\left(\frac{\partial V}{\partial r_f}\right)_{Stressed} = -\phi\omega T_2 Se^{-r_f T_2}\left[N_2\left(-\mu_{X_1 X_2 S}\phi\omega\left(X_2 - \sigma\sqrt{T_1}, \phi d_+; \omega\sqrt{T_1/T_2}\right)\right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.)\right]$$

Where, $\frac{\partial V}{\partial r_f} = -\phi\omega T_2 Se^{-r_f T_2} N_2\left(-\phi\omega\left(X_2 - \sigma\sqrt{T_1}, \phi d_+; \omega\sqrt{T_1/T_2}\right)\right)$.

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $\left(\frac{\partial V}{\partial r_f}\right)_{Stressed}$: positive, negative, high, low, close

and so on).We can find the **OPTIMUM** values of **RHO CLASS 1 and 2** in the same way as in the case of **DELTA CLASS 1 and 2** above.

2.3.5 Proposed Jameel's Stressed VEGA Models

Class 1:

$$\left(\frac{\partial V}{\partial \sigma}\right)_{Stressed} = Se^{-r_f T_2}\left[\begin{array}{l}\sqrt{t}\, n\left(X_2 - \sigma\sqrt{T_1}\right)\left(N\left(\mu_{X_1}\mu_{X_2}\mu_S \phi d_+^\tau\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right) \\ + \omega\sqrt{T_2}\, n\,(d_+)\left(N\left(\mu_{X_1}\mu_{X_2}\mu_S\left(-\phi\omega\frac{X_2\sqrt{T_2} + \sqrt{T_1} d_-}{\sqrt{\tau}}\right)\right) \pm \sigma_{X_1} f_1(.) \pm \sigma_{X_2} f_2(.) \pm \sigma_S f_3(.)\right)\end{array}\right]$$

Class 2:

$$\left(\frac{\partial V}{\partial \sigma}\right)_{Stressed} = Se^{-r_f T_2}\left[\begin{array}{l}\sqrt{t}\, n\left(X_2 - \sigma\sqrt{T_1}\right)\left(N\left(\mu_{X_1 X_2 S}\phi d_+^\tau\right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.)\right) \\ + \omega\sqrt{T_2}\, n\,(d_+)\left(N\left(\mu_{X_1 X_2 S}\left(-\phi\omega\frac{X_2\sqrt{T_2} + \sqrt{T_1} d_-}{\sqrt{\tau}}\right)\right) \pm \sigma_{X_1 X_2 S} f_1(.) \pm \sigma_{X_1 X_2 S} f_2(.) \pm \sigma_{X_1 X_2 S} f_3(.)\right)\end{array}\right] \text{Where,}$$

$$\frac{\partial V}{\partial \sigma} = Se^{-r_f T_2}\left[\sqrt{t}\, n\left(X_2 - \sigma\sqrt{T_1}\right) N\left(\phi d_+^\tau\right) + \omega\sqrt{T_2}\, n\,(d_+) N\left(-\phi\omega\frac{X_2\sqrt{T_2} + \sqrt{T_1} d_-}{\sqrt{\tau}}\right)\right].$$

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $\left(\frac{\partial V}{\partial \sigma}\right)_{Stressed}$: positive, negative, high, low, close

and so on).We can find the **OPTIMUM** values of **VEGA CLASS 1 and 2** in the same way as in the case of **DELTA CLASS 1 and 2** above.

2.4 Proposed Jameel's Advanced Stressed Exotic Options Pricing Models

2.4.1 Simple Chooser Options

Class 1:

$$^{Simple}Chooser_{Stressed} = Se^{-DT}\left[N\left(\mu_S\mu_K d_1\right) \pm \sigma_S f_1(.) \pm \sigma_K f_2(.)\right] - Ke^{-rT}\left[N\left(\mu_S\mu_K d_2 \pm \sigma_S f_1(.) \pm \sigma_K f_2(.)\right)\right]$$
$$- Se^{-DT}\left[N\left(-\mu_S\mu_K e_1\right) \pm \sigma_S f_1(.) \pm \sigma_K f_2(.)\right] + Ke^{-rT}\left[N\left(-\mu_S\mu_K e_2\right) \pm \sigma_S f_1(.) \pm \sigma_K f_2(.)\right]$$

Class 2:

$$^{Simple}Chooser_{Stressed} = Se^{-DT}\left[N\left(\mu_{SK} d_1\right) \pm \sigma_{SK} f_1(.) \pm \sigma_{SK} f_2(.)\right] - Ke^{-rT}\left[N\left(\mu_{SK} d_2 \pm \sigma_{SK} f_1(.) \pm \sigma_{SK} f_2(.)\right)\right]$$
$$- Se^{-DT}\left[N\left(-\mu_{SK} e_1\right) \pm \sigma_{SK} f_1(.) \pm \sigma_{SK} f_2(.)\right] + Ke^{-rT}\left[N\left(-\mu_{SK} e_2\right) \pm \sigma_{SK} f_1(.) \pm \sigma_{SK} f_2(.)\right]$$

Where, $^{Simple}Chooser = Se^{-DT} N\left(d_1\right) - Ke^{-rT} N\left(d_2\right) - Se^{-DT} N\left(-e_1\right) + Ke^{-rT} N\left(-e_2\right)$

$$d_1 = \frac{\ln(S/K) + (r - D + 0.5\sigma^2)T}{\sigma\sqrt{T}} , \quad e_1 = \frac{\ln(S/K) + (r - D) + 0.5\sigma^2 t}{\sigma\sqrt{t}} , \quad d_2 = d_1 - \sigma\sqrt{T}$$

And $e_2 = e_1 - \sigma\sqrt{T}$ .These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $^{Simple}Chooser_{Stressed}$: positive, negative, high, low, close and so on).We can find the **OPTIMUM** values of **SIMPLE CHOOSER OPTIONS CLASS 1 and 2** in the same way as in the case of **COMPOUND OPTIONS CLASS 1 and 2** above.

2.4.2 Complex Chooser Options

Class 1:

$$
\begin{aligned}
^{Complex}Chooser_{Stressed} =\ & Se^{-dT_1}\left[N_2\left(\mu_S\mu_{K_1}\mu_{K_2}(x, y_1; \rho_1)\right) \pm \sigma_{K_1} f_1(.) \pm \sigma_{K_2} f_2(.) \pm \sigma_S f_3(.) \right] \\
& - K_1 e^{-rT_1}\left[N_2\left(\mu_S\mu_{K_1}\mu_{K_2}(x - \sigma\sqrt{T_1}, y_1 - \sigma\sqrt{T_1}; \rho_1)\right) \pm \sigma_{K_1} f_1(.) \pm \sigma_{K_2} f_2(.) \pm \sigma_S f_3(.) \right] \\
& - Se^{-dT_2}\left[N_2\left(\mu_S\mu_{K_1}\mu_{K_2}(-x, -y_2; \rho_2)\right) \pm \sigma_{K_1} f_1(.) \pm \sigma_{K_2} f_2(.) \pm \sigma_S f_3(.) \right] \\
& + K_2 e^{-rT_2}\left[N_2\left(\mu_S\mu_{K_1}\mu_{K_2}(-x + \sigma\sqrt{T_2}, -y_2 + \sigma\sqrt{T_2}; \rho_2)\right) \pm \sigma_{K_1} f_1(.) \pm \sigma_{K_2} f_2(.) \pm \sigma_S f_3(.) \right]
\end{aligned}
$$

Class 2:

$$
\begin{aligned}
^{Complex}Chooser_{Stressed} =\ & Se^{-dT_1}\left[N_2\left(\mu_{SK_1K_2}(x, y_1; \rho_1)\right) \pm \sigma_{SK_1K_2} f_1(.) \pm \sigma_{SK_1K_2} f_2(.) \pm \sigma_{SK_1K_2} f_3(.) \right] \\
& - K_1 e^{-rT_1}\left[N_2\left(\mu_{SK_1K_2}(x - \sigma\sqrt{T_1}, y_1 - \sigma\sqrt{T_1}; \rho_1)\right) \pm \sigma_{SK_1K_2} f_1(.) \pm \sigma_{SK_1K_2} f_2(.) \pm \sigma_{SK_1K_2} f_3(.) \right] \\
& - Se^{-dT_2}\left[N_2\left(\mu_{SK_1K_2}(-x, -y_2; \rho_2)\right) \pm \sigma_{SK_1K_2} f_1(.) \pm \sigma_{SK_1K_2} f_2(.) \pm \sigma_{SK_1K_2} f_3(.) \right] \\
& + K_2 e^{-rT_2}\left[N_2\left(\mu_{SK_1K_2}(-x + \sigma\sqrt{T_2}, -y_2 + \sigma\sqrt{T_2}; \rho_2)\right) \pm \sigma_{SK_1K_2} f_1(.) \pm \sigma_{SK_1K_2} f_2(.) \pm \sigma_{SK_1K_2} f_3(.) \right]
\end{aligned}
$$

Where, $\quad ^{Complex}Chooser = Se^{-dT_1} N_2(x, y_1; \rho_1) - K_1 e^{-rT_1} N_2\left(x - \sigma\sqrt{T_1}, y_1 - \sigma\sqrt{T_1}; \rho_1\right)$
$$- Se^{-dT_2} N_2(-x, -y_2; \rho_2) + K_2 e^{-rT_2} N_2\left(-x + \sigma\sqrt{T_2}, -y_2 + \sigma\sqrt{T_2}; \rho_2\right)$$

$$x = \frac{\ln\left(Se^{-d\tau}/Xe^{-r\tau}\right)}{\sigma\sqrt{\tau}} + \frac{1}{2}\sigma\sqrt{\tau} , \quad y_i = \frac{\ln\left(Se^{-dT_i}/K_i e^{-rT_i}\right)}{\sigma\sqrt{T_i}} + \frac{1}{2}\sigma\sqrt{T_i} \text{ and } \quad \rho_i = \sqrt{\frac{\tau}{T_i}} .$$

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $^{Complex}Chooser_{Stressed}$: positive, negative, high, low, close and so on).We can find the **OPTIMUM** values of **COMPLEX CHOOSER OPTIONS CLASS 1 and 2** in the same way as in the case of **COMPOUND OPTIONS CLASS 1 and 2** above.

2.4.3 Lookback Options

The Stressed **FixedLookback Options Call** Options Pricing Models are given by:

Class 1:

$$
\begin{aligned}
C_{LB(Stressed)} =\ & Se^{-dT}\left[N\left(\mu_S\mu_M(x + \sigma\sqrt{T})\right) \pm \sigma_S f_1(.) \pm \sigma_M f_2(.) \right] - Me^{-rT}\left[N\left(\mu_S\mu_M(x)\right) \pm \sigma_S f_1(.) \pm \sigma_M f_2(.) \right] \\
& + \frac{S}{B}\left(e^{-rT}(S/M)^{-B}\left[N\left(\mu_S\mu_M(y + B\sigma\sqrt{T})\right) \pm \sigma_S f_1(.) \pm \sigma_M f_2(.) \right] - e^{-dT}\left[N\left(\mu_S\mu_M(y)\right) \pm \sigma_S f_1(.) \pm \sigma_M f_2(.) \right] \right)
\end{aligned}
$$

Class 2:

$$
\begin{aligned}
C_{LB(Stressed)} =\ & Se^{-dT}\left[N\left(\mu_{SM}(x + \sigma\sqrt{T})\right) \pm \sigma_{SM} f_1(.) \pm \sigma_{SM} f_2(.) \right] - Me^{-rT}\left[N\left(\mu_{SM}(x)\right) \pm \sigma_{SM} f_1(.) \pm \sigma_{SM} f_2(.) \right] \\
& + \frac{S}{B}\left(e^{-rT}(S/M)^{-B}\left[N\left(\mu_{SM}(y + B\sigma\sqrt{T})\right) \pm \sigma_{SM} f_1(.) \pm \sigma_{SM} f_2(.) \right] - e^{-dT}\left[N\left(\mu_{SM}(y)\right) \pm \sigma_{SM} f_1(.) \pm \sigma_{SM} f_2(.) \right] \right)
\end{aligned}
$$

The Stressed **FixedLookback Options Put** Options Pricing Models are given by:

Class 1:

$$P_{LB(Stressed)} = -Se^{-dT}\left[N\left(\mu_S \mu_M\left(-x-\sigma\sqrt{T}\right)\right)\pm\sigma_S f_1(.)\pm\sigma_M f_2(.)\right]+Me^{-rT}\left[N\left(\mu_S\mu_M(-x)\right)\pm\sigma_S f_1(.)\pm\sigma_M f_2(.)\right]$$

$$-\frac{S}{B}\left(e^{-rT}(S/M)^{-B}\left[N\left(\mu_S\mu_M\left(-y-B\sigma\sqrt{T}\right)\right)\pm\sigma_S f_1(.)\pm\sigma_M f_2(.)\right]-e^{-dT}\left[N\left(\mu_S\mu_M(-y)\right)\pm\sigma_S f_1(.)\pm\sigma_M f_2(.)\right]\right)$$

Class 2:

$$P_{LB(Stressed)} = -Se^{-dT}\left[N\left(\mu_{SM}\left(-x-\sigma\sqrt{T}\right)\right)\pm\sigma_{SM} f_1(.)\pm\sigma_{SM} f_2(.)\right]+Me^{-rT}\left[N\left(\mu_{SM}(-x)\right)\pm\sigma_{SM} f_1(.)\pm\sigma_{SM} f_2(.)\right]$$

$$-\frac{S}{B}\left(e^{-rT}(S/M)^{-B}\left[N\left(\mu_{SM}\left(-y-B\sigma\sqrt{T}\right)\right)\pm\sigma_{SM} f_1(.)\pm\sigma_{SM} f_2(.)\right]-e^{-dT}\left[N\left(\mu_{SM}(-y)\right)\pm\sigma_{SM} f_1(.)\pm\sigma_{SM} f_2(.)\right]\right)$$

Where, the Fixed Lookback Call and Fixed Lookback Put are respectively given by:

$$C_{LB} = Se^{-dT}N\left(x+\sigma\sqrt{T}\right)-Me^{-rT}N(x)+\frac{S}{B}\left(e^{-rT}(S/M)^{-B}N\left(y+B\sigma\sqrt{T}\right)-e^{-dT}N(y)\right)$$

And $P_{LB} = -Se^{-dT}N\left(-x-\sigma\sqrt{T}\right)+Me^{-rT}N(-x)-\frac{S}{B}\left(e^{-rT}(S/M)^{-B}N\left(-y-B\sigma\sqrt{T}\right)-e^{-dT}N(-y)\right)$

$B = \frac{2(r-d)}{\sigma^2}$, $x = \frac{\ln(S/M)+\left((r-d)-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$ and $y = \frac{-\ln(S/M)-\left((r-d)+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$.

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $C_{LB(Stressed)}$ and $P_{LB(Stressed)}$: positive, negative, high, low, close and so on).We can find the **OPTIMUM** values of **FIXEDLOOKBACK OPTIONS CALL AND FIXEDLOOKBACK OPTIONS PUT CLASS 1 and 2** in the same way as in the case of **COMPOUND OPTIONS CLASS 1 and 2** above.

2.4.4 Barrier Options

The Stressed Down – and – in Call Barrier Options Pricing Models are given by:

Class 1:

$$^{Down-and-In}C_{Stressed} = Se^{-qt}(B/S)^{2\lambda}\left[N\left(\mu_S\mu_K y\right)\pm\sigma_S f_1(.)\pm\sigma_K f_2(.)\right]$$

$$-Ke^{-rt}(B/S)^{2\lambda-2}\left[N\left(\mu_S\mu_K\left(y-\sigma\sqrt{T}\right)\right)\pm\sigma_S f_1(.)\pm\sigma_K f_2(.)\right]$$

Class 2:

$$^{Down-and-In}C_{Stressed} = Se^{-qt}(B/S)^{2\lambda}\left[N\left(\mu_{SK} y\right)\pm\sigma_{SK} f_1(.)\pm\sigma_{SK} f_2(.)\right]$$

$$-Ke^{-rt}(B/S)^{2\lambda-2}\left[N\left(\mu_{SK}\left(y-\sigma\sqrt{T}\right)\right)\pm\sigma_{SK} f_1(.)\pm\sigma_{SK} f_2(.)\right]$$

Where,

$$^{Down-and-In}C = Se^{-qt}(B/S)^{2\lambda}N(y)-Ke^{-rt}(B/S)^{2\lambda-2}N\left(y-\sigma\sqrt{T}\right)$$, $\lambda = \frac{r-q+r^2/2}{\sigma^2}$ and

$y = \frac{\ln\left[B^2/(SK)\right]}{\sigma\sqrt{T}}+\lambda\sigma\sqrt{T}$. B is the barrier level and assumed to be lower than the initial stock price.These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $^{Down-and-In}C_{Stressed}$: positive, negative, high, low, close and so

on).We can find the **OPTIMUM** values of **DOWN – AND – IN CALL BARRIER OPTIONS CLASS 1 and 2** in the same way as in the case of **COMPOUND OPTIONS CLASS 1 and 2** above.

2.4.5 Cash or Nothing Options:

Class 1:

$$CN_{Stressed} = Xe^{-rT}\left[N\left(\mu_X\left(\phi x - \phi\sigma\sqrt{T}\right)\right) \pm \sigma_X f_1(.)\right]$$

Class 2:

$$CN_{Stressed} = Xe^{-rT}\left[N\left(\mu_X\left(\phi x - \phi\sigma\sqrt{T}\right)\right) \pm \sigma_X f_1(.)\right]$$

Where, $CN = Xe^{-rT}N\left(\phi x - \phi\sigma\sqrt{T}\right)$

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $CN_{Stressed}$: positive, negative, high, low, close and so on).We can find the **OPTIMUM** values of **CASH OR NOTHING OPTIONS CLASS 1 and 2** in the same way as in the case of **COMPOUND OPTIONS CLASS 1 and 2** above.

2.4.6 Gap Options

Class 1:

$$Gap_{Stressed} = \phi Se^{-dT}\left[N\left(\mu_s\mu_x\phi x\right) \pm \sigma_s f_1(.) \pm \sigma_x f_2(.)\right]$$
$$- \phi Xe^{-rT}\left[N\left(\mu_s\mu_x\left(\phi x - \phi\sigma\sqrt{T}\right)\right) \pm \sigma_s f_1(.) \pm \sigma_x f_2(.)\right]$$

Class 2:

$$Gap_{Stressed} = \phi Se^{-dT}\left[N\left(\mu_{SX}\phi x\right) \pm \sigma_{SX} f_1(.) \pm \sigma_{SX} f_2(.)\right]$$
$$- \phi Xe^{-rT}\left[N\left(\mu_{SX}\left(\phi x - \phi\sigma\sqrt{T}\right)\right) \pm \sigma_{SX} f_1(.) \pm \sigma_{SX} f_2(.)\right]$$

Where, $Gap = \phi Se^{-dT}N\left(\phi x\right) - \phi Xe^{-rT}N\left(\phi x - \phi\sigma\sqrt{T}\right)$ and $x = \dfrac{\ln\left(\dfrac{Se^{-dT}}{Xe^{-rT}}\right)}{\sigma\sqrt{T}} + \dfrac{1}{2}\sigma\sqrt{T}$

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $Gap_{Stressed}$: positive, negative, high, low, close and so on).We can find the **OPTIMUM** values of **GAP OPTIONS CLASS 1 and 2** in the same way as in the case of **COMPOUND OPTIONS CLASS 1 and 2** above.

2.4.7 Asset Exchange Options

Class 1:

$$C_{Stressed} = S_1 e^{-d_1 T}\left[N\left(\mu_{S_1}\mu_{S_2} x\right) \pm \sigma_{S_1} f_1(.) \pm \sigma_{S_2} f_2(.)\right]$$
$$- S_2 e^{-d_2 T}\left[N\left(\mu_{S_1}\mu_{S_2}\left(x - \Sigma\sqrt{T}\right)\right) \pm \sigma_{S_1} f_1(.) \pm \sigma_{S_2} f_2(.)\right]$$

Class 2:

$$C_{Stressed} = S_1 e^{-d_1 T}\left[N\left(\mu_{S_1 S_2} x\right) \pm \sigma_{S_1 S_2} f_1(.) \pm \sigma_{S_1 S_2} f_2(.)\right]$$
$$- S_2 e^{-d_2 T}\left[N\left(\mu_{S_1 S_2}\left(x - \Sigma\sqrt{T}\right)\right) \pm \sigma_{S_1 S_2} f_1(.) \pm \sigma_{S_1 S_2} f_2(.)\right]$$

Where, $C = S_1 e^{-d_1 T} N(x) - S_2 e^{-d_2 T} N(x - \Sigma\sqrt{T})$, $\quad x = \dfrac{\ln\left(\dfrac{S_1 e^{-d_1 T}}{S_2 e^{-d_2 T}}\right)}{\Sigma\sqrt{T}} + \dfrac{1}{2}\Sigma\sqrt{T}$ and $\Sigma = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$.

These are the general FORMS of the proposal, however, we can use COMBINATORICS to obtain different possible Combination of TERMS and SIGNS (to achieve different values of $C_{Stressed}$: positive, negative, high, low, close and so on). We can find the **OPTIMUM** values of **ASSET EXCHANGE OPTIONS CLASS 1 and 2** in the same way as in the case of **COMPOUND OPTIONS CLASS 1 and 2** above.

2.5 Proposed Theorem (Lam's Theorem)

Let $\left(N(d_i), N_2(a_i, b_i; c_i), ..., N_n(a_i, b_i, ..., c_i; d_i)\right)$, $i = 1, 2, ..., n$ such that $n + m \in N$ are the number of terms and m is the number of repeated NORMAL probability type of an Exotic Options pricing model M_{Normal} whose probabilities are in this form, where $N_n(a_i, b_i, ..., c_i; d_i)$ is a Multivariate Normal Distribution. Let s and $f(s, \mu, \sigma, \xi)$ be the underlying stock return and **BEST FITTED** fat-tailed probability distribution of the underlying stock returns of company **A** respectively. Let $y_1, y_2, ..., y_n$ be the underlying returns of M_{Normal} **STRIKES** (Independent Variables) excluding underlying stock return. Let $f_1(y_1, \mu, \sigma, \pi)$, $f_2(y_2, \mu, \sigma, \pi), ..., f_n(y_n, \mu, \sigma, \pi)$ be **BEST FITTED** fat-tailed probability distribution of the underlying M_{Normal} **STRIKES** (Independent Variables). Let $\left(\mu_{y_1}, \mu_{y_2}, ..., \mu_{y_n}\right)$ and $\left(\sigma_{y_1}, \sigma_{y_2}, ..., \sigma_{y_n}\right)$ be vectors of respective Geometric Return of the Arithmetic Means of the macroeconomic indicators plus research company underlying stock returns (and **STRIKES** returns) and the respective Geometric volatility of the volatilities of the macroeconomic indicators plus research company underlying stock returns (and **STRIKES** returns). Define μ_S and σ_S as above then each of the **COMPONENTS** of $\left(N(d_i), N_2(a_i, b_i; c_i), ..., N_n(a_i, b_i, ..., c_i; d_i)\right)$, $i = 1, 2, ..., n$ in M_{Normal} can be contractionally and expansionally STRESSED using Jameel's stress methods as:

$$\left\{\begin{array}{l}\left(N\left(\mu_s\mu_{y_1}...\mu_{y_n}d_i\right) \pm \sigma_s f(s, \mu, \sigma, \xi) \pm \sigma_{y_1} f_1(y_1, \mu, \sigma, \pi) \pm ... \pm \sigma_{y_n} f_n(y_n, \mu, \sigma, \pi)\right), \\ \left(N_2\left(\mu_s\mu_{y_1}...\mu_{y_n}(a_i, b_i; c_i)\right) \pm \sigma_s f(s, \mu, \sigma, \xi) \pm \sigma_{y_1} f_1(y_1, \mu, \sigma, \pi) \pm ... \pm \sigma_{y_n} f_n(y_n, \mu, \sigma, \pi)\right), ..., \\ \left(N_n\left(\mu_s\mu_{y_1}...\mu_{y_n}(a_i, b_i, ..., c_i; d_i)\right) \pm \sigma_s f(s, \mu, \sigma, \xi) \pm \sigma_{y_1} f_1(y_1, \mu, \sigma, \pi) \pm ... \pm \sigma_{y_n} f_n(y_n, \mu, \sigma, \pi)\right)\end{array}\right\}, i = 1, 2, ..., n$$

Optimum at:

$$\left\{\begin{array}{l}\left(N(d_i) \pm \sigma_s f(s, \mu, \sigma, \xi) \pm \sigma_{y_1} f_1(y_1, \mu, \sigma, \pi) \pm ... \pm \sigma_{y_n} f_n(y_n, \mu, \sigma, \pi)\right), \\ \left(N_2(a_i, b_i; c_i) \pm \sigma_s f(s, \mu, \sigma, \xi) \pm \sigma_{y_1} f_1(y_1, \mu, \sigma, \pi) \pm ... \pm \sigma_{y_n} f_n(y_n, \mu, \sigma, \pi)\right), ..., \\ \left(N_n(a_i, b_i, ..., c_i; d_i) \pm \sigma_s f(s, \mu, \sigma, \xi) \pm \sigma_{y_1} f_1(y_1, \mu, \sigma, \pi) \pm ... \pm \sigma_{y_n} f_n(y_n, \mu, \sigma, \pi)\right)\end{array}\right\}, i = 1, 2, ..., n$$

for some $\sigma's = 1$ with (i) All $\mu_S = \mu_{y_1} = \mu_{y_2} = ... = \mu_{y_n} = 1$ and (ii) Some $\mu's = 1$.

OR

$$\left\{\begin{array}{l}\left(N\left(\mu_{sy_1...y_n}d_i\right) \pm \sigma_s f(s, \mu, \sigma, \xi) \pm \sigma_{sy_1...y_n} f_1(y_1, \mu, \sigma, \pi) \pm ... \pm \sigma_{sy_1...y_n} f_n(y_n, \mu, \sigma, \pi)\right), \\ \left(N_2\left(\mu_{sy_1...y_n}(a_i, b_i; c_i)\right) \pm \sigma_s f(s, \mu, \sigma, \xi) \pm \sigma_{sy_1...y_n} f_1(y_1, \mu, \sigma, \pi) \pm ... \pm \sigma_{sy_1...y_n} f_n(y_n, \mu, \sigma, \pi)\right), ..., \\ \left(N_n\left(\mu_{sy_1...y_n}d_i(a_i, b_i, ..., c_i; d_i)\right) \pm \sigma_s f(s, \mu, \sigma, \xi) \pm \sigma_{sy_1...y_n} f_1(y_1, \mu, \sigma, \pi) \pm ... \pm \sigma_{sy_1...y_n} f_n(y_n, \mu, \sigma, \pi)\right)\end{array}\right\}, i = 1, 2, ..., n$$

Optimum at:

$$\left(\begin{array}{l} \left(N\left(d_i\right) \pm \sigma_s f\left(s,\mu,\sigma,\xi\right) \pm \sigma_{sy_1 \cdots y_n} f_1\left(y_1,\mu,\sigma,\pi\right) \pm \ldots \pm \sigma_{sy_1 \cdots y_n} f_n\left(y_n,\mu,\sigma,\pi\right)\right), \\ \left(N_2\left(a_i,b_i;c_i\right) \pm \sigma_s f\left(s,\mu,\sigma,\xi\right) \pm \sigma_{sy_1 \cdots y_n} f_1\left(y_1,\mu,\sigma,\pi\right) \pm \ldots \pm \sigma_{sy_1 \cdots y_n} f_n\left(y_n,\mu,\sigma,\pi\right)\right), \ldots, \\ \left(N_n\left(a_i,b_i,\ldots,c_i;d_i\right) \pm \sigma_s f\left(s,\mu,\sigma,\xi\right) \pm \sigma_{sy_1 \cdots y_n} f_1\left(y_1,\mu,\sigma,\pi\right) \pm \ldots \pm \sigma_{sy_1 \cdots y_n} f_n\left(y_n,\mu,\sigma,\pi\right)\right) \end{array} \right), i = 1,2,\ldots,n$$

for some $\sigma_{y1,y2,y_n,S}$'s $=1$ with (i) All $\mu_{y_1 y_2 \ldots y_n.S} = 1$ and (ii) Some $\mu's = 1$. Also, there exists $c,d \in R^+$ such

that $c < d$ then $c < P_{i(Stressed)}^- \le P_i \le P_{i(Stressed)}^+ < b$ for each i.
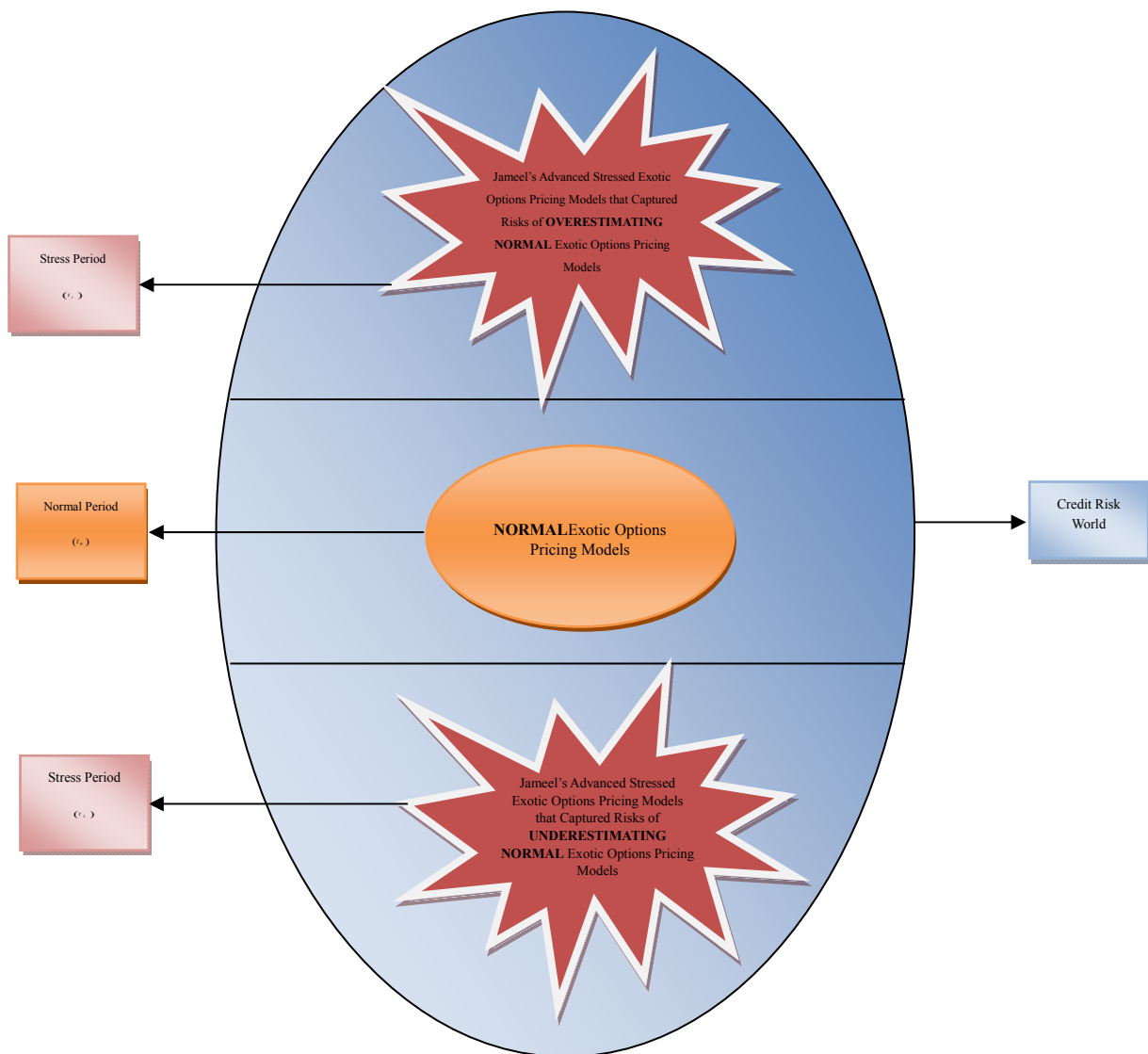
2.5.1 Jameel's Credit Risk World Diagram:

Figure 2. Kano ta Dabo's Diagram

3. Result and Discussion

All the proposed Jameel's Advanced Stressed Exotic Options Pricing Models are expected to adequately and efficiently work (more especially at the times of economic and financial recoveries or recessions) at their **OPTIMUM VALUES** stated in each case of the proposals based on the Jameel's Criterion and numerical results established and published in my previous papers.More so, the Models are expected to dramatically increases the markets **CONFIDENCE** and drastically decreases the markets **RISKS**.Jameel's Advanced Stressed Exotic Options Pricing Methods can be directly applied in other stochastic fields of **SCIENCES, ARTS, SOCIAL AND MANAGEMENT SCIENCES, MEDICINE AND ENGINEERING** to capture chaotic situations or to increase the power of the existing models' sensitivities to change.

4. Conclusion

The existing Exotic options pricing models underestimates (overestimates) prices especially at the times of economic and financial recoveries or recessions just because of their simple Normality Assumption and failure to incorporate underlying assets returns probability distributions. However, based on the research findings, the author improved the existing models by incorporating fat – tailed effects of the underlying assets returns of exotic derivative product in question using Jameel's Criterion and Jameel'sContractional and Expansional Stress Methods to enable them capture potential black swans.

For the sake of practitioners, it is believe that the existing exotic options pricing models out rightly underestimates (overestimates) exotic products prices especially at the stress periods to extent in which Nassim N. Taleb and EspenGararder Hang (2011) wrote a paper entitled: ' Option Traders use (very) Sophisticated Heuristic, never the Black – Scholes – Merton Formula', they stated that the formula is 'fragile to jumps and tail events' just because of its Normality Assumption thereby it is one of the major factors that contributed to the late 2007 – 2008 economic and financial crises (thanks to the Black – Scholes – Merton generous invention). In view of the above, all the proposed Jameel's Advanced Stressed exotic options pricing models class 1 and 2 presented in this paper, will be more reliable, robust, sophisticated, holistic and extraordinary, providing better approximations, increasing the probabilities of high losses and above all have the ability to precisely traces the trajectories of the past and future economic and financial crises related to exotic options pricing.

Finally, for the sake of future research direction, the models can be improved further to capture more vital information using more macroeconomic indicators and models' independent variables than ordinarily only $S, X_1 \text{ and } X_2$ or $S, K_1 \text{ and } K_2$.

CreditMetrics™ (1997) stated that "**We remind our readers that no amount of sophisticated analytics will replace experience and professional judgment in managing risks.** CreditMetrics™ is nothing more than a high-quality tool for the professionalrisk manager in the financial markets and is not a guarantee of specific results."

"If a seatbelt does not provide perfect protection, it still makes sense to wear one, it is better to wear a seatbelt than to not wear one". It is better off improving Exotic Options PricingModels to incorporate **fat – tailed effects** than not.

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