What Do Teacher Candıdates Know About the Limits of the Sequences?

Abdulkadir Tuna[1,*], Abdullah Cagri Biber[1] & Samet Korkmaz[2]

[1]Faculty of Education, Department of Mathematics Education, Kastamonu University, Kastamonu, Turkey

[2]Institute of Science, Mathematics Education Doctoral Program, Kastamonu University, Kastamonu, Turkey

*Correspondence: Faculty of Education, Kastamonu University, Kastamonu, Turkey. Tel: 90-366-280-3325. E-mail: atuna@kastamonu.edu.tr

Received: July 6, 2019 Accepted: August 22, 2019 Online Published: August 25, 2019

doi:10.5430/jct.v8n3p132 URL: https://doi.org/10.5430/jct.v8n3p132

Abstract

The aim of this study is to investigate the concept knowledge about the limits of sequences of mathematics teacher candidates. Research is a case study in which qualitative methods are adopted. The first phase of the study was conducted with a total of 45 teacher candidates taking the course of Analysis III. At this stage, the "Limit Knowledge Test in Sequences", which was developed by the researchers to investigate the concept knowledge of the sequence concept, was used as a data collection tool. At this stage, "Limit Knowledge Test in Sequences", which was developed by researchers to investigate the limit concept knowledge in sequences of the teacher candidates, included 2 open-ended problems were used as data collection tool. In the second phase of the research, individual interviews were made with 8 teacher candidates selected from the sample to conduct in-depth research. Content analysis was used to analyze the obtained data. As a result of the analysis of the data, significant shortcomings were found in the knowledge of the concept of the limit topic in the sequences of teacher candidates. It is seen that the candidates do not know the importance of the concept of "accumulation point", which is indispensable to convergence.

Keywords: mathematics education, mathematics teacher candidates, mathematics subject knowledge

1. Introduction

One of the main elements of the education of the teacher candidates is the subject content knowledge. The deficiencies in the subject knowledge constitute an obstacle to an effective teaching of mathematics (Halim & Meerah, 2002; Smith, 1999). Subject content knowledge is beyond having knowledge of a certain area. Subject content knowledge requires knowledge of basic concepts and principles. Subject content knowledge goes beyond having knowledge of a certain area and requires that the basic concepts and principles related to the subject to be taught are known and to be able to relate them to other topics (Cimer, 2008). The mathematics course in which conceptual understanding is important includes abstract, complex, and hierarchical concepts (Nesbit, 1996). Nevertheless, these concepts become more abstract and complex in the progressive class levels. As a consequence of this situation, some difficulties are encountered in teaching and learning mathematical concepts. One of the sub-areas of mathematics, the analysis topics are among the topics students encounter with difficulties in the teaching and learning of mathematical concepts (Cetin, Dane, & Bekdemir, 2012). The first concept that comes to mind in analysis is the concept of limit. The concept of limit has a very great importance in terms of being able to understand many important concepts such as continuity, series, derivative and integral (Arslan & Celik, 2013). One of the topic that limit concept is strongly related is the sequences. A is a non-empty set and a function that matches each element of the set of positive integers to an element from the set A is called a sequence. In other words, the sequence is a special type of function that that corresponds to a value for each positive integer (Balci, 2008). If we take a sequence (2/3, 3/4, 4/5,…), it is seen that the greatest value that elements of this sequence can reach is 1 (Bozkurt, 2013). As it is here, the value of the elements of the convergent sequence does not exceed a certain value (Adams, 2012). Similarly, as the value of the elements of the $(1/n^2)$ sequence increases, it approaches zero and as the value of n increases, the elements converge to a certain value, so this sequence is convergent. Determining whether sequences are convergent is important to know the behavior of the elements of the sequence. Just at this point, the importance of the limit concept emerges in the subject of the sequences. Along with the different ways of examining the convergence of sequences, whether a sequence is convergent or not is directly related to the limit of that sequence (Bozkurt, 2013).

Limit is the basis for mathematical concepts in other analytical topics, such as the convergence of sequences, therefore many researches have been made about this concept (Ozmantar & Yesildere, 2015). Studies on this subject show that students have conceptual misconceptions about the concept of limit in terms of "the highest value that can be reached", "a limit that should not be exceeded", "a value never reached" (Akbulut & Isik, 2005; Basturk & Donmez, 2011; Hitt & Lara, 1999; Szydlik, 2000). Akgun and Duru (2007) found that students had difficulty in learning about the limits of the sequences in their study conducted by second year students studying in science teacher education. When the related literature is examined, it appears that there are studies about the convergence of the sequences, but it is seen that a large part of these studies are for developing or compiling teaching methods related to the subject, whereas there are limited studies about examining the concept knowledge of teacher candidates (Alcock & Simpson, 2004, 2005; Berge, 2006; Doruk & Kaplan, 2013). Przenioslo (2005), in his work on the concept of convergence in sequences, has stated that the use of a specially designed set of problem and discussion questions on the subject allows students to develop their concept knowledge. Apart from the study by Doruk and Kaplan (2013) of mathematics teacher candidates examining proof evaluation skills on the concept of convergence of sequences, it is noteworthy that there is not any work on the convergence of the sequences in the national literature. Taking this into consideration, it is thought that this study, which aims to examine the knowledge of mathematics teacher candidates about the limit and convergence of sequences, will contribute to the literature. However, it is very important for teachers to have sufficient subject content knowledge when considering the importance of teaching profession, therefore it is very important to examine the conceptual knowledge of teacher candidates.

2. Method

This study aims to examine the conceptual knowledge of mathematics teacher candidates about the limit topic in the sequences, and it is a case study from the research types widely used in qualitative research approaches. The case study provides an in-depth examination of the researcher without intervention, the case study provides an opportunity to understand a case and offers the opportunity to find out about the impact of the case on the individual and society (Akar, 2016). The case study is used to describe and view the details that bring a case to the scene, to develop possible explanations about the case and to evaluate the case (Gall, Borg, & Gall, 1996).

2.1 Sample

Two different sampling methods were used in the selection of participants in the study. The study was first carried out with a total of 45 teacher candidates who were studying in the 3rd grade of Mathematics Teaching at a university in the north of Turkey in the fall semester of 2016-2017 academic year and who took the course of Analysis-III in the sequences-series topics. A simple random sampling method has been adopted in the selection of these teacher candidates (Gay, Mills, & Airasian, 2006). As they have already taken the Analysis I and Analysis II courses (Part 2), it is assumed that the participants have the necessary background information on the sequences topic. In the selection of the participants in the second phase of the study, criteria sampling method was adopted from purposeful sampling methods. The basic understanding of this sampling method is the study of all situations that meet a set of predetermined criteria. The mentioned criteria can be created by the researcher (Yildirim & Simsek, 2011). At this stage, individual interviews were made with the 8 teacher candidates who gave wrong answers to the questions on the data collection tool. In this way, it is aimed to carry out an in-depth examination of conceptual knowledge of teacher candidates.

2.2 Data Collection Tool

In this study, "Limit Knowledge Test in Sequences", which was developed by researchers to examine the concept knowledge of teacher candidates and had 2 open ended problems, was used as a data collection tool. Open-ended questions allow participants to express their thoughts freely, thus allowing the emergence of scientific thoughts and conceptual misconceptions (Bauer & Schoon, 1993). It is important that data collection tools consisting of open ended questions represent the suitability for the purpose of measurement and the extent to which it is intended to be measured. A review of the plans for the preparation of these tools may lead to a conclusion about the scope (Balci, 2015). Expert opinion has been consulted for the questions on the data collection tool in the study and it has come to the conclusion that the data collection tool is suitable for the purpose of the study. In addition, interviews were held with those who provided certain criteria among the teacher candidates participating in the study, in order to conduct an in-depth examination of the conceptual knowledge about the limit topic in the sequences.

2.3 Analysis of Data

Content analysis was used to analyze the data obtained in the study. According to Simon and Burstein (1985), content analysis is a systematic analysis of written and oral materials. It can be defined as the process of coding and digitizing the words and phrases of the individuals (Balci, 2015). The answers given by the prospective teachers who participated in the study to the questions in the "Limit Knowledge Test in the Sequences " were examined in three categories; correct, incorrect and empty. The data were independently coded and analyzed by two researchers who were experts in mathematics education. According to the reliability study performed later, the percentage of the correspondence between the researchers' coding was 83% (Miles & Huberman, 1994). The incompatible parts were passed through again, and a consensus was reached on them. Descriptive statistical techniques (percentage / frequency) were used in the analysis of the data obtained from the relevant test. The data obtained from the interviews conducted with the teacher candidates are interpreted by the phenomenological method. Phenomenographic method focuses not on individual but on differences in how individuals understand concepts, how they understand and interpret them (Marton & Booth, 1997). In other words, the phenomenological method focuses on the fundamental differences in the way the phenomenon is experienced, and the categorizations are related to each other by creating conceptual categories that reveal these differences (Cepni, 2007).

3. Findings

In this section, the findings of the analysis of the answers given by the prospective teachers to the questions in the " Limit Knowledge Test in the Sequences " and the interviews with the volunteer candidates are included. Due to the design of the study, some of the answers required are presented as examples. The answers that the candidates gave to the questions in the knowledge test were categorized as "right", "wrong" and "empty" and the findings were evaluated according to the order of the questions. The answers given to the questions in the knowledge test prepared to see how the teacher candidates define the limit in the sequences are analyzed in this section and the findings are given in Table 1 and Table 2.

Table 1. Findings of Answers to the First Question

Criteria	1. Question			
	a	b	c	d
	f (%)	f (%)	f (%)	f (%)
Right	35 (78)	37 (82)	35 (78)	26 (58)
False	6 (13)	5 (11)	4 (9)	9 (20)
Null	4 (9)	3 (7)	6 (13)	10 (22)

In the a-article of the first question of the knowledge test used in the research, "What is the convergent sequence? Identify.", it was found that 78% (n = 35) of the candidates were correct, 13% (n = 6) gave wrong definitions and 9% (n = 4) left this question empty. Here, the high rate of correct answers is remarkable. Many of the right answerers of this question have emphasized that for the convergent sequence, the general term of the sequence must have a limit value, or the general term converges to a value. Those, who responded wrongly to this question have argued that for convergent sequence, the limit of the general term of the sequence should be "0". Some examples of incorrect responses from candidates are given in Figure 1.

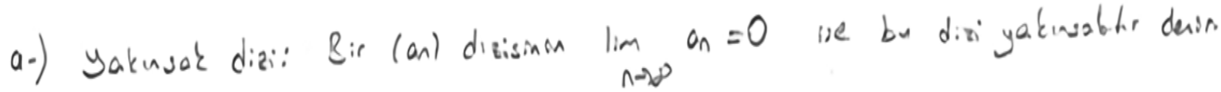

Figure 1-a.

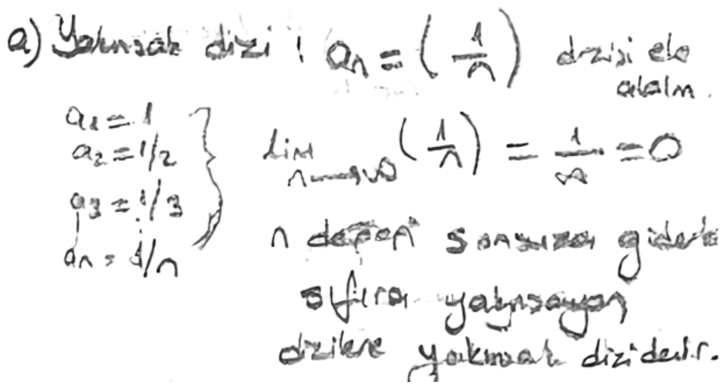

Figure 1-b.

The b-item of the first question was, "Show the limit of the sequence is 2". Here, 82% of the candidates (n = 37) answered correctly, the percentage of those who gave wrong answers and those who did not was 18% (n=8). This question holds both the theoretical knowledge (limit definition) and the operational knowledge (indicating that the limit of the sequence is 2 out of the way of defining the limit), but the success rate of the candidates is very high (82%). An example of correct answers is given in Figure 2. In this example, the candidate found a positive integer $n_0(\varepsilon)=5/\varepsilon$ such that $|a_n - a| < \varepsilon$ would be $n > n_0$ for $\forall \varepsilon > 0$. Here $n_0(\varepsilon)$ will change depending on the number ε.

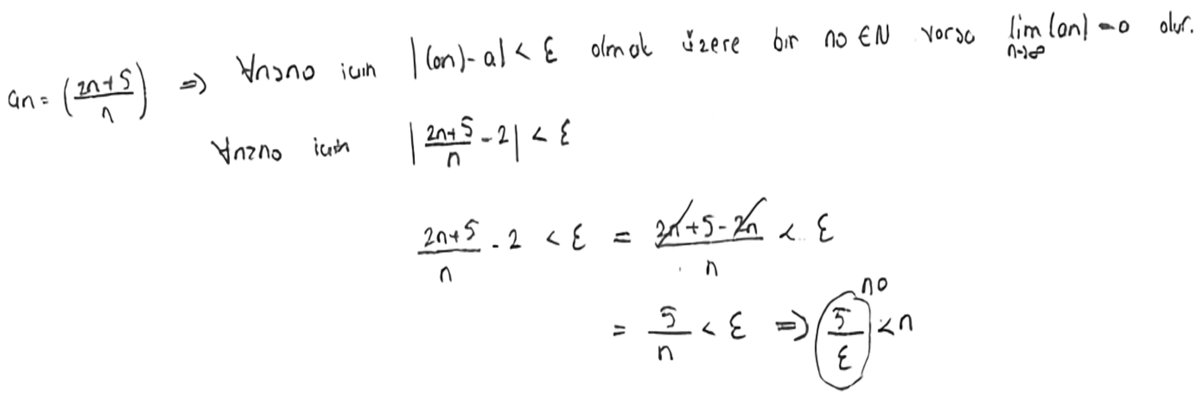

Figure 2.

In the first question, 78% (n = 26) of the candidates responded correctly to the third item " Write the terms of the sequence that are outside of $\varepsilon = 1/2$'s neighbors of 2." Here again, the high success rate is striking. In the question, a solution based on purely processing is required and it can be said that the candidates are not forced to this question because similar questions are solved in the lessons. Some examples from the correct answers given by the candidates are given in Figure 3 below.

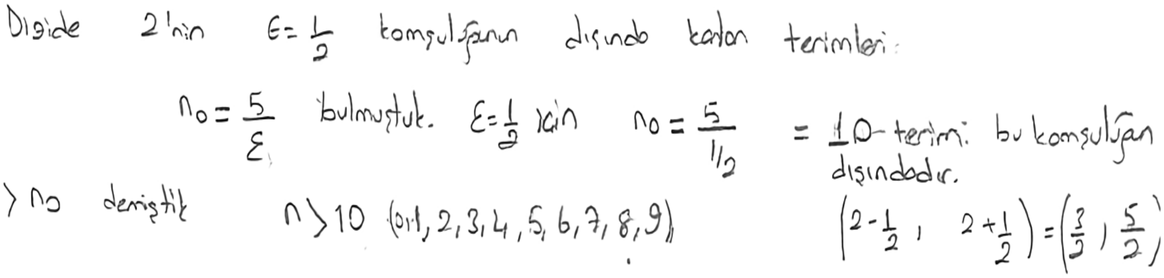

Figure 3-a.

$$a_n = \left(\frac{2n+5}{n}\right) \Rightarrow |a_n - a| < \varepsilon$$

$$\left|\frac{2n+5}{n} - 2\right| < \frac{1}{2} \qquad \rightarrow \frac{5}{n} < \frac{1}{2}$$

$$\left|\frac{2n+5-2n}{n}\right| < \frac{1}{2} \qquad 10 < n$$

↳ 1,2,3,4,5,6,7,8,9,10 (10 terimi komşuluğu dışında kaldı)

Figure 3-b.

The last item of the question is "Draw the graph of the given sequence and specify the limit on the graph." In this question, the success rate (58%) seems to fall surprisingly. The ratio of those who can not draw the graph correctly or the ones who can not show the limit correctly is 20% and the rate of those who do not solve this question is 22%. It is also remarkable that there are many teacher candidates who leave the question blank. Some examples of incorrect responses from candidates are given in Figure 4.

c) $a_n = \left(\frac{2n+5}{n}\right)$

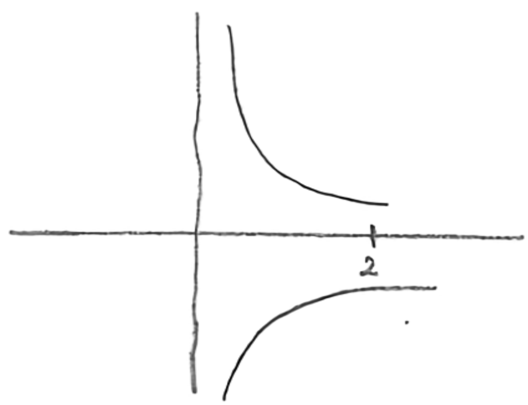

Figure 4-a.

The interview made with the teacher candidate who answered as in Figure 4-a is given below.

Researcher: Could you tell me how you draw the graph of the given sequence in this question?

Teacher Candidate: I drew down because the sequence was decreasing.

Researcher: Would you describe the graph you were drawing?

Teacher Candidate: On the graph, I showed that the elements of the sequence (showing the graph here) are converge to 2. (In the drawing of the candidate in Figure 4-a, the point to which the sequence converges is shown on the x-axis.)

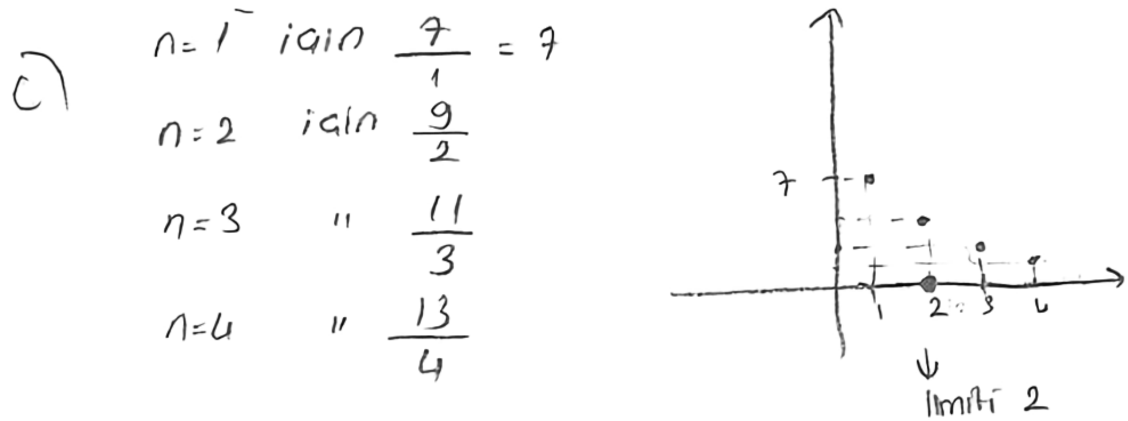

Figure 4-b.

The interview made with the teacher candidate who answered as in Figure 4-b is given below.

Researcher: Could you tell me how you draw the graph of the given sequence in this question?

Teacher Candidate: I showed values for n=1, n=2, n=3, ... and I showed their images in the coordinate system, then I showed that the sequence is converge to 2.

Researcher: Could you explain, why you pointed out 2 on the x-axis as "limit 2" in the graph you are drawing?

Teacher candidate: I realized I was wrong, I apologize. This (which means number 2) must be on the y-axis.

Here, it is seen that the teacher candidates have trouble in drawing the graph of the sequence and in showing the number that the sequence has converged on the graph. During the interviews, it was understood that the first candidate drawed the given sequence as though it were a continuous function. Moreover, the fact that both candidates show the point of convergence on the x-axis of the sequence shows, that candidates can not establish the input-output relation with the function concept. Because the value to which the sequence converges is determined by "output" and must be on the y-axis.

Table 2. Findings of Answers to the Second Question

Criteria	2. Question	
	a	b
	f (%)	f (%)
Right	24 (53)	
False	16 (36)	41 (91)
Null	5 (11)	4 (9)

The second question of the knowledge test consists of two chapters. In the question 2-a, “Calculate the limit for the sequence, that the generic term is $(a_n)=(\sin n/n)$. “, 53% (n = 24) of the candidates answered correctly, 36% (n = 16) answered incorrectly and 11% (n = 5) left the question blank. This question, together with being a question that requires fully operational knowledge, it is seen that the wrong answerers usually confuse the limit of the general term with the expression $\lim\limits_{n \to 0} (\sin x/x) = 1$ or they consider the expression as ∞/∞ ambiguity and then apply L'Hospital to solve it. Some examples of incorrect answers given by candidates to this question are given in Figure 5.

i) $\lim_{n\to\infty} \frac{\sin n}{n} = 1$ olur.

Figure 5-a.

i) $\lim_{n\to\infty}\left(\frac{\sin n}{n}\right) = \lim_{n\to\infty}\frac{\sin\infty}{\infty} = \frac{\infty}{\infty}$ belirsizliği

$= \lim_{n\to\infty}\frac{(\sin n)'}{n'} = \frac{1.\cos n}{1} = \underline{1}$ olur.

Figure 5-b.

In the second case of the second question, "Calculate the limit $\lim_{n\to 1}((n^2-1)/(1-n))$ for a sequence which generic term is $(n^2-1)/(1-n)$" it was seen that none of the candidates could give the correct answer. While 91% (n = 41) of the candidates answered incorrectly, the rate of those who left this question was 9% (n = 4). The lack of a correct answer to this question is a condition that requires an in-depth examination of the causes. When the answers given by the candidates were examined, it was seen that all of the solutions are the process of taking the normal limit. A few examples from the answers given by the candidates and the interviews with the candidates, who make these sample solutions, are given below.

$\lim_{n\to 1}\left(\frac{n^2-1}{1-n}\right) = \frac{0}{0}$ belirsizliği

$\lim_{n\to 1}\frac{2n}{-1} = -2$ olur.

Figure 6-a.

The interview made with the teacher candidate who answered as in Figure 6-a is given below.

Researcher: Could you briefly explain how you solve this question?

Teacher Candidate: As this limit is an uncertainty of $0/0$, I have obtained a derivative from L'Hospital.

Researcher: In your solution, you've got $(-2n)$ after applying L'Hospital rule. So what is the limit of a sequence with a generic term $(a_n)=(-2n)$?

Teacher candidate: (After pausing for a while) Infinite.

Investigator: Why infinite?

Teacher Candidate: Because there was only a limit for $n\to\infty$ in the sequences.

Researcher: Why are there limits only for $n \to \infty$ in sequences?

Teacher Candidate: I do not know.

$$\lim_{n\to 1}\left(\frac{n^2-1}{1-n}\right)=\lim_{n\to 1}\left(\frac{\overset{-1}{\cancel{(n-1)}}(n+1)}{\cancel{1-n}}\right)=\lim_{n\to 1}(-n-1)=-1-1=\boxed{-2}$$

Figure 6-b.

The interview made with the teacher candidate who answered as in Figure 6-b is given below.

Researcher: Could you briefly explain how you solve this question?

Teacher Candidate: I made simplification because the expression given here is undefined for n = 1. Then, I wrote down the point, where the limit was sought.

Researcher: Then, can we say that the general term of the sequence is $(a_n)=(-n-1)$?

Teacher candidate: Yes, because I just did a simplification.

Investigator: What is the limit of a sequence that has a generic term $(a_n)=(-n-1)$?

Teacher Candidate: The limit for $n \to 1$ is -2.

The above interview examples provide important information about how teacher candidates perceive the limit concept in the sequences. It is understood that the candidates do not separate the limit subject in sequence, from the limit subject in the functions. It is seen that the candidates do not pay attention to the concept of the accumulation point while focusing on the limit operation, they are focused only on the operations. Because no real number is the accumulation point of Z^+, the definitionsset of the sequences, so no teacher candidate could think that a sequence would not have any real number limit.

4. Conclusion

In this study, the conceptual knowledge of teacher candidates about the limit topic in the sequences is examined. According to the research findings, it is possible to reach the following conclusions about the concept knowledge of the candidates in terms of limits in the sequences;

Candidates have generally defined the concept of "convergent sequence" (78%) as "a limit of the general term of the sequence" or " the general term of the sequence, has to converge to a number ". However, incorrect respondents (13%) have argued remarkably that the limit of the general term of the sequence should be "0". This is confused with the theory that "the limit of the general term of convergent series is zero". Similarly, it has also been expressed in various studies that students confused sequence and serial concepts (Ciltas, 2011; Schwarzenberger & Tall, 1978).

In the second option, "Show the limit of $((2n+5)/n)$ sequence is 2", 82% of the candidates were able to give a correct answer by finding a positive integer as $n_0(\varepsilon)=5/\varepsilon$ such that $|a_n-a|<\varepsilon$ is $n>n_0$ for $\forall\varepsilon>0$. Later on, the candidates showed a similar success (78%) in the case of " Write the terms of given sequences that are outside the neighbors of 2". Although these solutions seem to be heavily weighted on the theoretical knowledge, it is not surprising that the success rate is high because it is very similar to the solutions made in the lessons. More precisely, it is not wrong to say that the prospective teachers are doing the solutions by heart. It is possible to understand this from the downside of the correct answers (58%), that the candidates give to the last option of the first question. In the final question, "Draw the graph of the given sequence and specify the limit on the graph", it is observed that the candidates have experienced difficulties while the candidates pointed to the point where the sequence converges. In the final question, "Draw the graph of the given sequence and specify the limit on the graph", it is observed that the candidates have experienced difficulties when they showed the point where the sequence converged on the charts they plotted. Most of the candidates specify the limit of the given sequence in the graphs plotted in the definition set, not in the value set. In the case of the limit topic, the problems experienced in the relationship between the definition set of functions and the set of values, have been expressed in various studies (Bezuidenhout, 2001; Cetin, Dane, & Bekdemir, 2012; Jordaan, 2005; Szydlik, 2000).

In the a-option of the second question of the knowledge test, 53% of the candidates gave the correct answer to the question "Calculate the limit of the sequence with the general term $(a_n) = (\sin n/n)$". This question can actually be seen as a simple limit-taking process, because teacher candidates are more familiar with the limit-taking process than high school and Analysis-2 courses. However, many of the candidates who responded to this question confused this limit with the expression of $\lim_{x\to 0}(\sin x/x) = 1$ or candidates have seen the expression as ∞/∞ ambiguities and have tried to find a solution by applying L'Hospital. Likewise, Ciltas (2011) used the same question in his work and stated that the candidates relate this question to the limit rules in the functions and insist on it even in their interviews. The difficulties that students have with regard to limit and limit concepts have been expressed in various researches (Akbulut & Isik, 2005; Basturk & Donmez, 2011; Jordaan, 2005; Szydlik, 2000; Tall & Vinner, 1981; Tall, 1993).

In the second option of the second question, "Calculate the limit $\lim_{n\to 1}((n^2-1)/(1-n))$ for an sequence with a generic term $(n^2-1)/(1-n)$", it was seen that none of the candidates could give the correct answer. When the answers given by the candidates were examined, it was seen that all of the solutions consisted of taking the normal limit. In the interviews conducted to investigate the causes of the answers to this question in more detail, it was seen that none of the candidates could give satisfactory answers about why the limit of a sequence could not be searched around a real number. In interviews conducted to investigate the causes of the answers to this question, it was seen that none of the candidates could give satisfactory answers as to why an sequence could not be searched around a real number. This shows that candidates do not know the importance of the point of accumulation, which is essential for the concept of convergence. Since no real number is the accumulation point of a set of positive integers, which is the definition set of sequences, it is pointless to search for the limit of a sequence around any real number (Kadioglu & Kamali, 2009). While candidates give high answers to all the questions in the knowledge test, it is very thought-provoking that no one answered correctly in this question, in which there is a completely conceptual knowledge. It is understood that the candidates are focused on the mathematical operation skill completely, and the candidates do not give the necessary importance to the conceptual knowledge. Likewise, Cetin, Dane and Bekdemir (2012) emphasize that university students can not correctly define the concept of " accumulation point" in a study they have done, but that the lack of knowledge of this concept may be the reason for the difficulties and misconceptions experienced in limit and other concepts. Similarly, in other studies, it has been shown that that lack of information about the accumulation point leads to misconceptions about the limit (Basturk & Donmez, 2011; Przenioslo, 2004; Swinyard & Lockwood, 2007).

As a result of this study in which teacher candidates investigate how they understand the limit concept in the sequence, it is thought that the subject of the sequences should be designed and processed in a suitable teaching method in the lessons. Especially the differences between sequence and function should be emphasized in lessons. Because the sequence is known as a special function, it is necessary to emphasize the properties of the sequences such as the graphs of the sequences and convergence in the sequences. Lesson activities should be prepared taking this into consideration. The significance of the concept of the accumulation point, and the meaning of the accumulation point for the limit concept, must be explained in detail in the lessons. In addition, similarities and differences between concepts of series and sequences should be explained with appropriate examples.

References

Adams, M. R. (2017). *Sequences and series: An introduction to mathematical analysis.* Retrieved 02.01.2017 from http://math.uga.edu/~lyall/3100/sands.pdf.

Akar, H. (2016). Durum calismasi. In A. Saban & A. Ersoy (Eds.), *Egitimde nitel arastirma desenleri* (pp. 113-149). Ankara: Ani Yayincilik.

Akbulut, K., & Isik, A. (2005). Limit kavraminin anlasilmasinda etkilesimli ogretim stratejisinin etkinliginin incelenmesi ve bu surecte karsilasilan kavram yanilgıları. *Kastamonu Egitim Dergisi*, *13*(2), 497-512.

Akgun, L., & Duru, A. (2007). Misunderstandings and difficulties in learning sequence and series: A case study. *Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education*, *11*(1), 75-85.

Alcock, L., & Simpson, A. (2004). Convergence of sequences and series: Interactions between visual reasoning and the learner's beliefs about their own role. *Educational Studies in Mathematics, 57,* 1-32. https://doi.org/10.1023/B:EDUC.0000047051.07646.92

Alcock, L., & Simpson, A. (2005). Convergence of sequences and series 2: Interactions between nonvisual reasoning and the learner's beliefs about their own role. *Educational Studies in Mathematics*, *58*, 77-100.

https://doi.org/10.1007/s10649-005-2813-5

Arslan, S., & Celik, D. (2013). Zor sanilan iki kavram: Limit ve sureklilik. In I. O. Zembat, M. F. Ozmantar, E. Bingolbali, H. Sandir & A. Delice (Eds.), *Tanimlari ve tarihsel gelisimleriyle matematiksel kavramlar* (pp. 463-487). Ankara: Pegem Akademi.

Balci, A. (2015). *Sosyal bilimlerde arastirma: Yontem, teknik ve ilkeler* (11th ed.). Ankara: Pegem Akademi.

Balci, M. (2008). *Genel Matematik* (5th ed.). Ankara: Balci Yayinlari.

Basturk, S., & Donmez, G. (2011). Matematik ogretmen adaylarinin limit ve sureklilik konusuyla ilgili kavram yanilgilari. *Necatibey Egitim Fakultesi Elektronik Fen ve Matematik Egitimi Dergisi*, *5*(1), 225-249.

Bauer, M., & Schoon, I. (1993). Mapping variety in public understanding of science. *Public Understanding of Science*, *2*(2), 141-155. https://doi.org/10.1088/0963-6625/2/2/004

Berge, A. (2006). Convergence of numerical sequences – commentary on "The vice: some historically inspire and proof generated steps to limits of sequences" by R.P. Burn. *Educational Studies in Mathematics*, *61*, 395-402. https://doi.org/10.1007/s10649-006-8754-9

Bozkurt, A. (2013). Diziler: Belli bir kurala gore sirali listeler. In I. O. Zembat, M. F. Ozmantar, E. Bingolbali, H. Sandir & A. Delice (Eds.), *Tanimlari ve tarihsel gelisimleriyle matematiksel kavramlar* (pp. 489-499). Ankara: Pegem Akademi.

Bezuidenhout, J. (2001). Limits and continuty: some conceptions of first-year students. *International Journal of Mathematics Education in Science and Techonolgy*, *32*(4), 487-500. https://doi.org/10.1080/00207390010022590

Cepni, S. (2007). *Arastirma ve proje calismalarina giris*. Trabzon: Celepler Yayincilik.

Cetin, O. F., Dane, A., & Bekdemir, M. (2012). Yigilma noktasi kavrami ve kullanimi. *Necatibey Egitim Fakultesi Elektronik Fen ve Matematik Egitimi Dergisi*, *6*(2), 217-233.

Ciltas, A. (2011). *Dizi ve seriler konusunun matematiksel modelleme yoluyla ögretiminin ilkogretim matematik ogretmeni adaylarının ogrenme ve modelleme becerileri uzerine etkisi.* Unpublished doctoral dissertation, Ataturk University, Erzurum.

Cimer, A. (2008). Ogretmen egitiminde gelismeler ve uygulamalar. In N. Saylan (Eds.), *Egitim bilimine giris*. Ankara: Ani Yayincilik.

Doruk, M., & Kaplan, A. (2013). Ilkogretim matematik ogretmeni adaylarinin dizilerin yakinsakligi kavrami uzerine ispat degerlendirme becerileri. *Egitim ve Ogretim Arastirmalari Dergisi, 2*(1), 231-240.

Gall, M., Borg, W., & Gall, J. P. (1996). *Educational research an introduction* (6th ed.). USA: Longman Publisher.

Gay, L. R., Mills, G. E., & Airasian, P. (2006). *Educational Research*: *Competencies for analysis and applications*. Columbus: Merrill Greenwood.

Halim, L., & Meerah, S. M. (2002). Science teachers' pedagogical content knowledge and its influence on physics teaching. *Research in Science and Technological Education*, *20*(2), 215-227. https://doi.org/10.1080/0263514022000030462

Hitt, F., & Lara, H. (1999). Limits, continuity and discontinuity of functions from two points of view: That of the teacher and that of the stıdent. In L. Bills (Eds.), *Proceedings of the British Society for Research into Learning Mathematics* (Vol. 19).

Jordaan, T. (2005). *Misconceptions of the limit concept in a mathematics course for engineering students*. Unpublished Master of Science Dissertation, University of South Africa.

Kadioglu, E., & Kamali, M. (2009). *Genel matematik*. Kultur Egitim Vakfi (5. Baski). Erzurum, 30-42.

Marton, F., & Booth, S. (1997). *Learning and awareness*. Hillsdale, NJ: Lawrence Erlbaum Ass.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative Data Analysis: An Expanded Sourcebook* (2nd ed.). Newbury Park, CA: Sega.

Nesbit, T. (1996). What counts? Mathematics education for adults. *Adult Basic Education, 6*, 69-83.

Ozmantar, M. F., & Yesildere, S. (2015). Limit ve sureklilik konularinda kavram yanilgilari ve cozum arayislari. In M. F. Ozmantar, E. Bingolbali & H. Akkoc (Eds.), *Matematiksel kavram yanilgilari ve cozum onerileri* (pp.

181-221). Ankara: Pegem Akademi.

Przenioslo, M. (2004). Images of the limit of function formed in the course of mathematical studies at the university. *Educational Studies in Mathematics*, *55*, 103-132.

Przenioslo, M. (2005). Introducing the concept of convergence of a sequence in secondary school. *Educational Studies in Mathematics*, *60*, 71-93. https://doi.org/10.1007/s10649-005-5325-4

Schwarzenberger, R. L. E., & Tall, D.O. (1978). Conflicts in the learning of real numbers and limits. *Mathematics Teaching*, *8*(2), 44–49.

Simon, J. L., & Burstein, P. (1985). *Some Principles of Measurement*. New York: Random House.

Smith, D. C. (1999). Changing our teaching: The role of pedagogical content knowledge in elementary science. In J. Gess-Newsome & N. G. Lederman (Eds.), *Pedagogical content knowledge and science education*. Dordrecht: Kluwer Academic Publishers.

Swinyard, C., & Lockwood, E. (2007). Research on students' reasoning about the formal definition of limit: An evolving conceptual analysis. *In Proceedings of the 10th annual conference on research in undergraduate mathematics education*. San Diego, CA: San Diego State University.

Szydlik, J. E. (2000). Mathematical beliefs and conceptual understanding of the limit of a function. *Journal for Research in Mathematics Education*, *31*(3), 258-276. https://doi.org/10.2307/749807

Tall, D., & Vinner, S. (1981). Concept images and concept definitions in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*, 151-169. https://doi.org/10.1007/BF00305619

Tall, D. (1993). *Students' difficulties in calculus, Proceedings of Working Group 3 on Students' Difficulties in Calculus*. ICME-7, Quebec, Canada, 13-28.

Yildirim, A., & Simsek, H. (2011). *Sosyal bilimlerde nitel arastirma yontemleri* (8th ed.). Ankara: Seckin Yayincilik.