

# Mathematics Undergraduate Student Teachers' Conceptions of Guided Inductive and Deductive Teaching Approaches

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## Abstract

This contribution aimed at developing an understanding of student teachers' conceptions of guided discovery teaching approaches. A cross-sectional survey design involving eleven secondary mathematics teachers who had enrolled for an in-service mathematics education degree was used to address the research question: What are undergraduate student teachers' conceptions of deductive and inductive teaching approaches? Task-based interviews were used in conjunction with oral interviews as settings for unravelling students' conceptions of the two teaching approaches. Drawing in part from Ausbel's learning theory and Tall's notion of a met-before, the study also aimed at assessing the students' level of grasp of fundamental limitation of empirical explorations despite many benefits associated with them such as helping in identifying patterns, use in formulation and communicating of conjecture, and providing insights on what needs to be solved. Verbatim transcriptions from follow up interviews and textual data from task based interviews were subjected to directed content analysis to infer meaning about students' conceptions of guided teaching approaches. Qualitative data analysis using in part Robert Moore's notion of concept usage uncovered conceptual limitations that include inconsistencies in student teachers' definitions of the teaching approaches, use of specific examples instead of arbitrary mathematical objects in illustrating analytic teaching. Limitations identified should be given attention by mathematics educators in order to increase understanding of the approaches among teachers. Research studies into factors contributing to limitations identified can go a long way in improving the teaching and learning of school mathematics.

**Keywords:** *guided teaching; empirical arguments; deductive and inductive instruction; mathematical proof and proving; deductive and inductive thinking*

## 1. Introduction

This paper is situated in an overall view of mathematics as a problem solving human endeavour and the broad notion of mathematical knowledge for teachers (Stylianides and Ball, 2008, p. 308). The teaching of mathematics places significant demands on both the subject matter and pedagogical knowledge of secondary school mathematics teachers (Jones, 1997). Research has shown that school mathematics teaching has largely adopted an inductive style (Harel and Sowder, 1998; Stylianides, p. 1). The lead author has observed from personal experience during high school teaching and even presently as he interacts with peers that inductive teaching approach enjoys considerable prominence. Inductive teaching uses empirical arguments (inductive explorations or use of specific examples) to derive generalizations and has several benefits such as high retention of mathematical facts and identification of patterns (CadawalladerOlsker, 2011; Harel and Sowder, 1998; Morselli, 2006, p. 6). Yet empirical explorations do not offer complete and conclusive evidence about the truth of a mathematical generalization. This limitation of empirical verifications applies to statements that require proof by deductive argumentation (Stylianides, 2011, p. 2). The lead author's reading around the notion of mathematical proof made him wonder if secondary school teachers are aware of limitations of empirical explorations. Deductive thinking habits are supposed to be inculcated in learners through the guided deductive teaching approach that appears to be used sparingly in mathematics lessons at secondary school level (Harel and Sowder, 2007). Further, formal deductive reasoning is the desired level of thinking

at all scholastic levels that has the potential to be a vehicle for deep mathematical learning in secondary mathematics (Iaanone and Inglis, 2011, p. 2). Hence, there is need for more studies into secondary school mathematics teacher competencies on the two learner-centred teaching approaches. This study aimed at contributing to the domain called pedagogical content knowledge (PCK) by (Shulman, 1987). A cross-sectional survey was adopted that involved in-service secondary mathematics teachers on professional development studies. Theories such as constructivism, Ausbel's learning theory, together with Tall (2008)'s notion of a met- before provided analytic lenses for the study. What follows next is a description of guided discovery teaching and other related theoretical stances about deductive and inductive teaching approaches.

## 2. Theoretical Considerations

In guided discovery the teacher designs a sequence of activities that are meant to facilitate the discovery of mathematical principles, patterns and relationships by the learners. Two broad teaching and learning approaches in mathematics, the inductive discovery and the deductive discovery approaches, can be distinguished. The use of the term guided or directed discovery with the two teaching approaches points to the teacher's role in the instruction process, which is that of facilitating the discovery of patterns, connections, and relationships by the learners (Parker & Goicoechea, 2000; Prince and Felder, 2006). Guided discovery teaching approaches utilize learner-centred strategies and consequently they are heavily steeped in constructivism (Dengate and Lerman, 1995, p. 37). We briefly reflect on constructivism and its tenets, then consider suggestions on how content could be packaged in line with constructivist theory.

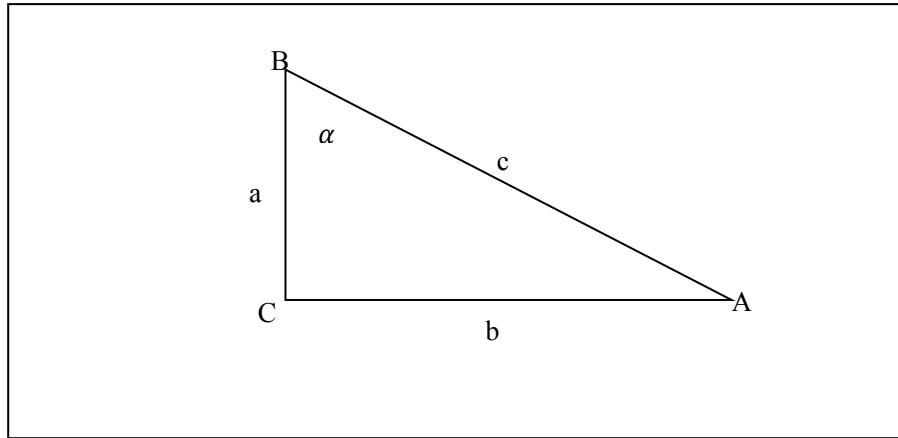
Constructivism posits that learners actively construct and reconstruct their own reality in an effort to make sense of their own experiences (Prince and Felder, 2006, pp. 3-5; Dengate and Lerman, 1995, p. 37). During the sense making process new information filters through the students' mental structures (schemata). This process of information filtering depends on students' met-befores, which is a collection of the student's prior knowledge, beliefs, prejudices, preconceptions and misconceptions (Prince and Felder, 2006; Tall, 2008). Strands of constructivism include cognitive constructivism first mainstreamed by Jean Piaget and later by a host of other researchers. According to cognitive constructivism learning is conceived in terms of the learner's reactions to his or her experiences. There is also social constructivism advanced initially by Lev Vygotsky. Here, there is much emphasis on the role of language, peers, and teachers in the construction of meaning from experiences. Sense making is collaboratively done. The ideas drawn from constructivism inform packaging of content when employing the two teaching approaches. This includes the need for teachers to consider learners' relevant prior knowledge and experiences when developing new knowledge. Presentation of content should be done in a manner that does not require abrupt or drastic changes in learners' cognitive models. New knowledge should allow learners to operate within their zone of proximal development (ZPD) (Wellings, 2003, p. 8). Students should be weaned from the practice of receiving knowledge from teachers. Instead, the learners should discover patterns, and extrapolate from material presented while the role of the teachers is to guide learners (Prince & Felder, 2006, p. 4). But what do the deductive and inductive teaching approaches entail?

Directed inductive teaching approach is whereby instruction begins with 'specifics,' typically a set of observations or experimental data. As the learners try to analyse and interpret the specific examples, specific scenarios with some hints and other assistance from the teacher, the learners then realize or discover the mathematical generalizations, rules, procedures and mathematical principles (Prince and Felder, 2006, p.1). Hence with inductive teaching mathematical relationships are built by learners as they quantitatively evaluate the generalizations in a proper subset of all possible cases (Stylianides, 2011, p. 1; Harel and Sowder, 1998, 2007). These quantitative evaluations may involve numeric tests or trials of given relationships and reflections on the specific examples (Morselli, 2006, p. 6). Several benefits are derived from these inductive explorations of specifics. Some of the benefits include building mathematical connections (Morselli, 2006), discovering of patterns, providing insights on what needs to be solved during problem solving (Harel and Sowder, 1998) and retention of mathematical facts (Prince and Felder, 2006).

Inductive thinking is the natural tendency to evaluate mathematical statements probabilistically. Inductive thinking habits have been shown to be well established among learners (CadawalladerOlsker, 2009). Once inculcated in learners inductive thinking habits they have been reported to be difficult to eradicate or undo (CadawalladerOlsker, 2009; Harel and Sowder, 1998). It was one of the goals of this study to find out the level of grasp of this well documented limitation of inductive teaching among in-service mathematics student teachers in spite of its aforementioned benefits. We now turn to deductive thinking.

A teacher employing deductive teaching approach would start with *theories* and guide learners in discovering the

generalizations (Prince and Felder, 2006). *Theories* here imply axioms, definitions, and previously proven results (generalizations) that are used to guide learners in deriving desired mathematical generalization(s). An appropriate exemplification of deductive teaching in school mathematics would be the derivation of one of the fundamental trigonometric identities such as  $\sin^2 \alpha + \cos^2 \alpha = 1$ . To derive the generalization the following theories are requisite.



**Figure 1.** The Pythagorean Theorem for Triangle ABC Right Angled at C.

It can be seen from Figure 1 that  $b^2 + a^2 = c^2$ . Letting angle  $ABC = \alpha$ , we obtain the following trigonometric ratios of sine and cosine of angle  $ACB$  as  $\sin \alpha = b/c$ ,  $\cos \alpha = a/c$ . Dividing the stated theorem by  $c^2$  and substituting squares of the trigonometric ratios in the equation obtained by dividing theorem expression by  $c^2$  yields the fundamental trigonometric identity. Emphasis is made here that with the deductive approach, as illustrated above, there is use of arbitrary mathematical objects (Haggarty, 1992; Kirkwood, 1992). For instance in this example there is use of arbitrary sides and angle dimensions of a right-angled triangle as opposed to use specific or particular examples as in the inductive approach. It is this distinguishing characteristic of deductive thinking that makes its results more generalizable than those obtained by inductive explorations of specific examples only. It was a goal of the study to determine the level of awareness of this fundamental distinction between the two approaches among the pre-service student teachers.

Some researchers (e.g., Prince and Felder, 2006, p. 1) equate deductive teaching to the lecture method of teaching which is often teacher-centred. They further suggest, or rather claim, that the deductive approach is less constructivist, and hence its association with a teacher-centred approach. We differ with that suggestion and argue instead that deductive teaching can be constructivist-oriented and also use a learner-centred approach depending entirely on how the mathematical knowledge is presented to the learner. Thus the way of presenting the subject matter determines the learner-centeredness or teacher-centeredness of a teaching approach and this is independent of the form of reasoning or argumentation involved which could then be said to be deductive or inductive or even abductive in nature. Formal deductive thinking in mathematics lessons is the desired level for learners to attain (Dewey in Stylianides, 2011, p. 1; Harel and Sowder, 1998). Morselli (2006, p. 2) emphasizes this point saying “this use of examples must be overcome to reach a formal proof.” The preference for increased deductive teaching over inductive teaching in mathematics education relates to the observation that inductive arguments do not provide complete and conclusive evidence about the truth of mathematical relationships. Our interest in examining secondary teachers’ conceptions of the two teaching approaches suggested the main research question:

### 3. Research Question

The overarching research question that guided the study was: *What are undergraduate student teachers’ conceptions of deductive and inductive discovery teaching approaches?*

The objectives underlying the research question were (a) to explore undergraduate student teachers’ conceptions of deductive and inductive teaching approaches, and (b) to assess student teachers’ abilities to construct proofs of mathematical statements using deductive and inductive justifications

#### 4. Situating the Study

Elementary teaching practices that promote or tolerate an understanding that mathematical generalizations are consequences of empirical explorations instil mental habits that significantly deviate from conventional understanding of mathematics (Stylianides, 2011, p. 1). Dewey in Stylianides (2011, p. 2) encourage educators to desist from such practices when he says:

Whatever preliminary teaching/ learning approach students engage in, it should not lead to inculcation of mental habits which later on have to be bodily displaced or uprooted to secure proper comprehension of a mathematical generalization or concept.

Other ideas related to student teachers' conceptions of the two approaches are drawn from Ausbel's theory of learning. The theory posits that learning takes place by assimilating new information into existing conceptual structures of the learner (Varghese, 2009). Closely linked to Ausbel's theory of learning is Tall's (2008) notion of met-before in his discussion of the three worlds of mathematics. Tall defines a met-before as the current existing mental structure of an individual based on prior experiences of that person. Tall suggests that previous learner experiences form connections in the brain which will then be consistent or inconsistent with the new learning.

An implication of these proposals (Ausbel's learning theory and Tall's idea of met-before) is that the shift from inductive explorations to the desired deductive thinking in mathematics learning at undergraduate level is difficult. This emanates from the observation that school mathematics teaching is largely inductive (CadawalladerOlsker, 2009; Harel and Sowder, 1998; Stylianides, 2011). Consequently, within the learners' conceptual frameworks, the natural tendency to derive generalizations through use of 'specifics' exist as a strong met-before which will be difficult to do away with as learners try to switch to deductive derivations of mathematical relationships. The study explored secondary school teachers' understanding of two teaching approaches, and as such the findings may help inject important ideas that can inform the teaching and learning of mathematical proof during teacher education and, perhaps, to some extent influence participating teachers classroom practice.

#### 5. Method

##### 5.1 Research Design

A cross-sectional survey was employed to gain insights into the nature of student teachers' understanding of the two teaching approaches as well as determining the manner in which student teachers employ deductive and inductive means to build mathematical generalizations. In other words, a snapshot of secondary school teachers' conceptions of the guided inductive and guided deductive teaching approaches (Lavrakas, 2008) with the intent to develop an appraisal of these conceptions and their characteristics (Setia, 2016). The survey study was also conducted with the intent to assess how inductive and deductive proving approaches are deployed in proving given theorems.

##### 5.2 Participants and Sampling

A total of eleven in-service secondary school mathematics teachers studying for a Bachelor degree in mathematics education were involved in the study. Purposive sampling—which is a non-probability sampling technique—was employed because it was considered strategic for a survey with a strong qualitative orientation as it aimed to generate insights on student teachers' conceptions of deductive and inductive teaching strategies. Each teacher participant in the study held a Diploma in Education with mathematics as a major subject from one of the secondary school teacher education colleges in Zimbabwe. Teacher preparation in Zimbabwean secondary teacher education colleges encompasses studies in pedagogical content knowledge in secondary school mathematics. It was therefore assumed that participants were familiar with the two teaching approaches explored, and also that mathematical generalizations included in the questionnaire were assumed to be within the student teachers' conceptual reach. Mathematical statements which students were required to prove were drawn from the Ordinary and Advanced curricula. Student teachers had covered concepts included in the task-based interview during their teacher education studies.

Written responses and interviews were used for data collection (Varghese, 2009, p. 3). Study participants were on Virtual Open Distance Learning (VODL) format run by one university of science education in Zimbabwe. The VODL programme for staff development of secondary teachers had two learning components. There was the Virtual component where learners receive tuition in the form modules and assignments through online services. The other component was the face to face teaching during school holidays at selected high schools conveniently located near students' work places. The study took place during face to face sessions. The four week long block release teaching period was too short to allow prolonged engagement with the student teachers and hence the decision to employ a cross-sectional survey design instead of a case study.

### 5.3 Data Generation Procedure

Data collection took place during second week of a four week long block release teaching period when pressure from other modules had eased. The aim was to assess student teachers' conceptions of the two teaching approaches as well as assessing their abilities to prove given theorems. The instrument comprised three sections. Section A elicited demographic data of participants for the sole purpose of ensuring participants had the prerequisite pedagogical content knowledge (PCK) for secondary school mathematics in Zimbabwe. This information was captured on the subsection written 'Qualification.' Section B of the questionnaire was meant to assess students' conceptions of deductive and inductive teaching approaches through written responses to an item that required the student teachers to distinguish between the two approaches to teaching and learning of mathematics. Two mathematical generalizations were examined. One was drawn from the Zimbabwe School Examination Council Ordinary level syllabus (ZIMSEC/ 4008) while the other generalization was from taken fundamental trigonometry studied in Zimbabwean Secondary Teacher Education Colleges.

The task-based interview comprised the following activities:

#### Section B

1. *Distinguish between inductive and deductive teaching in mathematics.*
2. *Describe how you would teach any one of the following generalizations using:*
  - (a) *Guided inductive teaching,*
  - (b) *Guided deductive teaching*

Generalization 1: *The angle subtended by an arc at the centre of a circle is twice the angle subtended at circumference.* Generalization 2:  $\sin^2 \alpha + \cos^2 \alpha = 1$

In the task-based interviews the student teachers described in written form how they would derive one of mathematical generalisations using the two teaching approaches (Housman and Porter, 2003; Varghese, 2009). During the ensuing interviewing phase, the students were asked to articulate with justification(s) how they would have proved the given statements. The intention of this second phase of the data collection process was to elicit expressions of students' abilities to construct proofs including their level of grasp of the fundamental limitation of inductive teaching, that is, use of specific examples to establish mathematical relationships (CadawalladerOlsker, 2009; Stylianides, 2011, p. 2). The third section of the data collection instrument consisted of interview questions whose formulation depended upon the kind of written responses to section B. The student teachers' articulations were audio-taped and then interview transcriptions were then produced by one of the three authors.

### 5.4 Data Analysis

After production of verbatim transcripts of the follow up interviews by one of the researchers (who is the first author), three data sets were then produced, one for each of the three researchers. A data set consisted of written responses to items in sections A and B of the task-based interviews and verbatim transcriptions of the follow up interviews to the written responses for each of the 11 students involved in the study. The 3 researchers then met and discussed matters related to the data analysis procedure. The analytic framework was discussed that include ideas related to whether a student teacher had used arbitrary elements or *theories* when describing how the prospective high school teacher would teach the generalization using either inductive or deductive teaching approaches. The researchers also agreed on the need to check whether students had resorted to empirical verifications in their descriptions of how they would derive the generalizations during teaching. During discussion of the analytic framework the researchers agreed on the need to check for consistency (or lack thereof) between verbatim transcriptions and written attempts by the student teachers. Hence, directed content analysis was deemed to be versatile and strategic by the researchers because it allowed them to immerse themselves in qualitative data using deductive codes derived from their interrogation of literature on deductive and inductive mathematical reasoning whilst they stayed open for inductive codes that could emerge from the data that were being analysed (Ndemo and Mtetwa, 2015).

Other ideas that informed part of the preparatory discussion of data analysis included Moore's (1994) notion of concept usage. Briefly, concept usage is defined as "the ways one operates with the concept in generating or using examples" (Moore, 1994, p. 252). A qualitative analysis of the student teachers' understandings of the two teaching approaches was based on the students' descriptions (meanings) of the two teaching approaches, ideas on use of the two approaches to establish mathematical generalisations with learners, and an assessment of their grasp of the limitation of empirical explorations in deriving mathematical relationships (Varghese, 2009). The researchers applied a qualitative content analysis technique called directed content analysis that allowed them to infer meaning about the

students' conceptions of two forms of mathematical reasoning from their written responses. The researchers applied content analysis independently to the written responses and verbatim transcriptions. Researchers then met to discuss emerging themes about students' conceptions of inductive and deductive approaches to teaching.

## 6. Results and Discussion

This section is organized into two main sub-sections according to the two research objectives. One of the main objectives was to explore students' conceptions of the guided deductive and inductive teaching approaches. Students' responses to Section B of the data collection instrument and follow up interviews gave data that had a focus on this objective. Researchers' inferences drawn from textual data are supported by in-vivo codes, that is, actual students' written responses and utterances from follow up interviews (Corbin and Strauss, 2008; Varghese, 2009). Results and discussion that relate to student teachers' understandings of the two forms of mathematical reasoning are described next.

### 6.1 Students' Conceptions of the Two Approaches

As intended, the data collected provided information on the students' conceptions of the two teaching approaches. Overall, the students' descriptions demonstrated lack of basic understanding of the two approaches. The majority of the student teachers' written responses were incorrect and revealed some inconsistencies with literature on guided inductive and deductive teaching strategies. To quote (using pseudonyms) some of responses on inductive teaching, one of the students, for example, said

This entails generalisation from a given example, that is, from general to particular [Gibson]

The response reveals some contradiction in the sense that the student claimed that inductive teaching uses "...a given example," which is true with regard to inductive explorations, but the student went on to write that with inductive discovery relationships are built from general to particular, which contrasts with literature on inductive reasoning. Inductive reasoning involves use of instantiations to build generalizations but the term "from general" by the student implies use of arbitrary elements, which is a key feature of deductive reasoning. This reflects a limitation in the students' knowledge of the teaching approaches. Other responses indicated a lack of clarity, for example,

Teacher demonstrates and moves together with students up to the final stage of the problem [Mike]

Teaching from unknown to known [Alves]

Once again, the students' written responses illustrate their *confusions*, that is, lack of basic understanding of deductive discovery teaching approach. It is important to note that when employing either the deductive or inductive approach to teaching it is possible that a "teacher demonstrates and moves together with students up to the final stage of the problem," as claimed by Mike, so it can be deduced he had not grasped the distinction between the two teaching approaches. Other responses that also revealed lack of clarity in students' grasp of the distinction between the two forms of mathematical reasoning include,

Teaching from known to unknown [Alves]

This entails using raw information, critically examine it, and come up with a formula or approach that can be used in other instances [Gibson]

Gibson's response revealed poor concept usage (Moore, 1994) by the student with regard to deductive reasoning because what Gibson refers to as "using raw information critically" shows some inconsistencies with definition of deductive thinking as shown by his attempt at deriving the generalization analytically (deductively) when he wrote,

Measure angle at centre and angle and measure angle at circumference [Gibson]

Measuring connotes empirical or inductive explorations (Harel and Sowder, 1998, 2007; Stylianides, 2011), pointing to the student teacher's *confusions* or weak understanding of the deductive teaching approach.

On a positive note, student teachers' responses revealed a good grasp of the teacher's role in the teaching and learning process. These were very much in line with constructivist views (Prince and Felder, 2008). To quote some of the responses:

...students are guided by the teacher to find a given formula [Maria]

Pupils take a leading role in deducing, may be a formula [Tecla]

Maria's and Tecla's responses are consistent with constructivist ideas that learners should actively engage in the sense making process during knowledge construction as opposed to receiving knowledge. The teacher's role in

guided teaching was clearly expressed by Maria as being that of guiding the knowledge construction process rather than that of transmitting mathematical facts.

### 6.2 Students' Inductive and Deductive Derivations of the Generalizations

Commenting on aspects of proof that a prover needs to be able to handle mentally and technically Selden and Selden (2009) posit that it is not important that a prover is able to articulate the formal rhetoric part—which actually captures the anticipated proof behaviour—but it is essential that a prover is able to execute this behaviour. Following Selden and Selden's comment, it was important to assess the student teachers' capabilities to implement the two teaching approaches in teaching mathematics at secondary school level. Hence, results of the students' attempts at deriving mathematical relationships inductively and analytically are discussed next.

With inductive explorations there was a tendency by the students to accept a few examples and in many cases one example as evidence of the truth of a mathematical generalisation. This finding corroborates finding from research studies by (Harel and Sowder, 1998; Stylianides, 2011) where students were found to accept a few specific examples, usually numeric trials, as conclusive evidence about the truth of a general mathematical statement. Closely linked to the use of few examples, or even a single example, was a limitation in the students' pedagogical knowledge of deductive teaching approach revealed through a case where the derivation of the generalization was conceived in terms of finding a solution to a particular problem as illustrated.

If angle  $ADB = 102$  (referring to angle at centre), Calculate angle  $x$  (referring to angle at circumference) [Tedijs]

Tedijs' effort points to a possible dominance of inductive explorations in school mathematics teaching in secondary school in Zimbabwe where a mathematical generalization was thought of in terms a single particular example without revealing the embedded underlying relationships between angle at centre and angle at circumference. It can therefore be inferred that because of such limitations in subject matter knowledge, Tedijs may have perhaps been resorting to a *drill and practice* teaching repertoire. Tedijs' attempt points to the importance of the interplay between subject content knowledge (SCK) and pedagogical content knowledge (Stylianides and Ball, 2008).

Another show of weak understanding from the teachers—all of whom had teaching experience of at least five years—was revealed by the students' use of specific examples in deductive teaching of mathematical generalizations. Deductive derivations employ arbitrary mathematical objects in building mathematical generalisations (Haggarty, 1992; Kirkwood, 1992). Amola's written response was a case in point that contradicted this assertion, that is, use of arbitrary elements in deducing relationships. Amola made use of specific known angles as illustrated.

Taking  $\alpha = 45$  degrees then,  $\sin\alpha = \frac{\sqrt{2}}{2}$  and  $\cos\alpha = \frac{\sqrt{2}}{2}$ , Squaring  
yields  $\sin^2 \alpha = \frac{1}{2}$  and  $\cos^2 \alpha = \frac{1}{2}$  and adding gives;  $\sin^2 \alpha + \cos^2 \alpha = 1$  [Amola]

Amola also considered the case when  $\alpha = 30$  degrees, repeated the process illustrated above and then concluded on the basis of the two specific angles that  $\sin^2 \alpha + \cos^2 \alpha = 1$ . Amola's proof attempt revealed limitation in his command of deductive mathematical reasoning as derivation of the trigonometric identity was conceived in terms of particular instantiations described. Amola's and Tedijs' efforts revealed that these student teachers were unaware of the fundamental limitation of use of empirical verifications in building mathematical generalizations. Amola employed a single example while Tedijs' proof attempts involved use of two referential mathematical objects. Tedijs' and Amola's deductive reasoning attempts conflict with the well documented fact that inductive explorations do not provide conclusive evidence about truth of mathematical statements (CadawalladerOlsker, 2009; Stylianides, 2011).

Finally, the students' responses show that some students conceived deductive teaching as being necessarily teacher centred. For example, although Maria's utterances during follow up interviews revealed that she was able to articulate the role of the teacher in guided deductive and inductive teaching, she went on to describe deductive teaching approach as being teacher centred as reflected in the following extract.

The weakness of this method is that pupils mainly depend on the teacher.

They do not have confidence to research on their own [Maria]

Maria's response is consistent with claims by Prince and Felder (2006) who equate deductive teaching to the lecture method. We have already disputed this claim in our engagement with the literature. We reiterate that Maria's utterances revealed her limited grasp of the deductive teaching approach which can be tailored to be student centred depending on how a teacher unpacks the knowledge. Maria's description of the teacher's role in constructivist teaching and its associated benefits were not consistent with her claims that deductive teaching is teacher centred.

## 7. Concluding Remarks

The study revealed some limitations in the student teachers conceptions of the inductive and deductive teaching approaches. The limitations include: inconsistencies or *confusions* in student teachers' definitions of the two teaching approaches, use of specific examples as opposed to use of arbitrary mathematical objects in analytic (deductive) development of mathematical relationships, and unawareness of the scope of applicability of empirical verifications of mathematical statements. The study also indicated that the student teachers conceived deductive teaching in terms of solving a particular problem. Since teachers' conceptions of a pedagogical instructional approach inevitably shape classroom practice (Jones, 1997; Varghese, 2009), it is important that such limitations in understanding receive attention from mathematics educators. A study that could account for such weak student understandings as discussed in this paper can also contribute to improvements in the teaching and learning of school mathematics.

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