

Impacts of COVID on University's Finances

A Vector Autoregressive Analysis in Quebec, Canada

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Abstract

Did universities benefit from the pandemic? Some did receive more funding than usual. We use vector autoregressive models to forecast both enrollment and public subsidies in a jurisdiction where public funding depends mostly on enrollment. Using unemployment as an established proxy for the impact of recessions on enrollment, we show that the recent COVID pandemic increases pressure on public subsidies. Further, we use our forecasts to decompose the current subsidies between long-term subsidies, recession induced subsidies, and additional funding. We find that the subsidies given during the pandemic were higher than what a typical recession would command for 8 universities out of 18.

Keywords: universities, enrollment, subsidies, forecasts, recession, pandemic

1. Introduction

The COVID-19 pandemic has had its toll on universities. They had to massively migrate classes online and adapt their practice to a new reality triggered by social distancing (Dennen, 2020). In parallel, the pandemic also brought about important changes in student enrollment. On the one hand, the pandemic caused massive changes in unemployment, leading several persons to consider higher education studies in the interim. On the other hand, students seeking a campus experience or preferring in-class curriculums deferred their studies to get an education closer to their expectations. These two effects, amongst many others, determine implicitly the final student demand in the context of a pandemic.

Enrollment can bear significant importance to determine university finances. Most jurisdictions that use an input-based funding formula rely on a measure of enrollment to transfer public funds to universities (Frølich, 2010). As such, changes in enrollment can have cumulative effects beyond tuition.

This paper uses empirical techniques to explore how university enrollment and revenues evolve with the forecasting horizon. Comparing a period of one to four years (henceforth mid-term) with a period of twenty years (henceforth long-term) allows us to assess the impact of mid-term factors on enrollment and finance. Aside from the range in years, the difference between forecasts relies on the use of unemployment and other variables as a proxy of mid-term effects on demand. Thus, by comparing the difference between these forecasted values for the next year (henceforth the short-term), one can infer the impact of unemployment on both funding and enrollment. Further, by comparing *realized values* for the current year, we can further refine the decomposition of effects and find the impact of other factors. In the case of public appropriations, we can deduce the likely change in funding induced by the current pandemic (compared to what would have been otherwise allocated).

Globally, we find that unemployment has a significant mid-term effect on enrollment on most universities, including aggregate systems of universities. By extension, when a funding formula is tied to enrollment, this surge in enrollment implies additional pressure for public appropriations. We also find significant differences between forecasted and realized appropriations in the short-term for several universities. Finally, we also find a great heterogeneity in the magnitude of responses: some universities show a far greater response to changes in the underlying explanatory variables than others. In particular, unemployment has more impact on larger universities. Combined, these three results provide novel insights on the behaviour of university systems in times of a pandemic, or any other disruptive economic event that translates into high unemployment. These results are detailed in more depth in the body of the

article and can be helpful for both funding planners and university managers alike.

Aside from behavioural insights, this article also develops a robust forecasting methodology based on historical trends in both funding and enrollment. It uses Vector autoregressive models (henceforth VARs) to simultaneously estimate the evolution of enrollment and public appropriations for mid-term forecasts. To our knowledge, this has never been done before. For long-term forecasts, the paper employs instrumental variable regressions (henceforth IV). Both techniques carry a similar statistical advantage. That is, it accounts for the potential endogeneity (interdependence) between public appropriations and enrollment. In jurisdictions where public funding is driven by enrollment, this is quite important: an unanticipated positive shock on enrollment should translate into an increase in public funding. In statistical terms, this means that the estimation error for public subsidies is dependent on the estimation error of enrollment. Using a statistical technique that accounts for such interdependence is useful both for a pure forecasting perspective, and to guarantee an absence of bias in the estimated relationship between enrollment and funding. By using these techniques, we thus present an approach reflecting the typical impact of the true relationship, in the sense of unbiased, between enrollment and funding. These approaches can be useful for practitioners having to forecast both their subsidies and their enrollment patterns.

Because of its availability and of our existing knowledge of the funding formula, we chose the province of Quebec, Canada as the underlying jurisdiction for our analysis (Ministère de l'Éducation et de l'Enseignement supérieur, 2020). The province has 18 independent degree granting universities, all having public funds as their main stream of revenues (see Table 1). Six universities have more than 20 000 full-time equivalent students (henceforth FTEs) in a wide variety of programs and tiers; two universities are engineering schools; six universities are regional flagships; two universities are small graduate schools with less than 800 FTEs; one is an online university; and one is a business school. Ten of those universities are members of the Quebec University system, which was one large public university at its inception in the 60's, in the spirit of many public university systems existing in other jurisdictions (CUNY, UC, UT, etc).

Table 1. Synthetic representation of the Québec university system

	Québec University System	Independent universities
Universities with more than 20,000 FTEs and a wide variety of programs.	Université du Québec à Montréal (UQAM)	Concordia University* McGill University* Université Laval Université de Montréal Université de Sherbrooke
Engineering schools	École des technologies supérieures (ETS) +	École polytechnique de Montréal.
Regional universities	Université du Québec en Abitibi-Témiscamingue (UQAT) Université du Québec en Outaouais (UQO) Université du Québec à Rimouski (UQAR) Université du Québec à Trois-Rivières (UQTR) Université du Québec à Chicoutimi (UQAC)	Bishop's University * +
Small vocational graduate schools	École nationale d'administration publique (ENAP) Institut national de recherche scientifique (INRS)	
Other	Téléuniversité du Québec (TÉLUQ) +	Hautes études commerciales (HEC)

Legend: Predominantly English Speaking Universities are marked with an asterisk (*) and undergraduate only universities are marked with a plus (+). Universities without an asterisk are predominantly french speaking and universities without a plus sign award graduate degrees. Source: Author's classification and Ministère de l'Enseignement supérieur (Éditeur officiel du Québec, 2020).

2. Literature Review

To our knowledge, there exist no research papers forecasting higher education institution's subsidies, nor papers combining forecasts on enrollment and finances, through the use of time series techniques. Below, we rely on two strands of the literature to situate the scope of our work. First, we look at papers forecasting enrollment through univariate time series methods. Second, we consider papers investigating the relationship between enrollment and unemployment, or recessions. We thus share an estimation paradigm with the first strand and the second strand explores how economic conditions may affect enrollment patterns. Although not within the scope of this review, it should be known that the use of the time series estimation paradigm for other types of financial forecasts is well established. One can look at Andersen, David, KreiB and Mikosch (2009) for a handbook on the matter.

A time series estimation paradigm is focused on predicting future values of one or multiple variables on the basis of its time trends and a few other variables (Hamilton, 1994). Conversely, it is not a paradigm seeking for multiple variables or causes. Although far from representing the complete model ecosystem, two workhorse models have a long tradition: the univariate autoregressive, integrated moving average framework (ARIMA) and the vector-autoregressive framework (VAR) (Box, Jenkins, Reinsel & Ljung, 2015). The ARIMA model and its variants focus on forecasting a single variable, while VARs seek to forecast more than one variable, and their chronologic interdependence, at the same time. Forecasting techniques applied to enrollment with pre-existing approaches to the time-series paradigm are also discussed in Gu'rin (1972), Shaw (1984), and Kwak, Brown and Schiederjans (1986).

To our knowledge, Marshall and Oliver (1979) are the first to study enrollment patterns through the use of autoregressive (AR) models. Their study decomposes enrollment at a given time by the semester of enrollment, each cohort being modeled as an AR process. They then forecast total enrollment in a given year as the sum of each forecast. Chen, Li and Hagerdorn (2019) also use seasonal ARIMA models to study enrollment patterns of international students in the midwest, American university. They characterize enrollment patterns through seasonal changes, and institutional variables such as the stringency to obtain American student visas and unemployment. Their final model shows that past enrollment predicts enrollment, as well as persistence in deviations of real values from the forecast. They also find a positive relationship between enrollment and unemployment. Chen (2008) and O'Bryant (1991) have also performed a similar analysis on different institutions. Qin, Shanks, Phillips and Bernard (2019) study how the structure of ARIMA models best fit real patterns and simulated ones. Through simulations, they find that a time-series model estimated on 20 observations outperforms models estimated on larger samples. They then find similar results on true patterns of enrollment. Their findings seem to be much related to the concept of overfitting (Hawkins, 2004). Closer to our work, Boes and Pflaumer (2006) use an ARIMA framework to predict student enrollment in Germany for a variety of universities, each with its own model. They underline the use of time-series models most notably to include margins of errors on forecasts, which is said to be missing from official German predictions (*id*).

Simultaneous forecasts through VARs are employed by Ewing, Beckert and Ewing (2010). They simultaneously predict enrollment patterns, economic growth and inflation. The authors find some interdependence between enrollment and economic patterns that has a Keynesian flavour. For instance, a surge in inflation leads to an increase in enrollment, and an economic downturn is also shown to increase enrollment patterns. The theoretical foundations for a countercyclical connection between enrollment and economic activity is however a feat of classic economic theories as well and has a well-established foundation (e.g. Dellas & Koubi, 2003; Dellas & Sakellaris, 2003). The idea is rooted in the opportunity cost of time. When unemployment is high, the opportunity cost of going to school is much lower by the absence of a paying alternative (e.g. a job). The converse is true during booms. Such relationship between recessions on enrollment patterns has been fairly well documented through both historical accounts (Barr & Turner, 2013) and empirical approaches (Witteveen, 2020; Smith, 2017). As argued in Alessandrini (2014), this can change the perspective on recessions, as they do allow for an accumulation of graduates that will turn productive once graduated. Following that logic, Sievertsen (2016) shows with individual level data that the unemployment rate increases graduation rates significantly and he further shows that part of this surge in enrollment comes from individuals who would not have enrolled otherwise. This effect is also shown to vary by ethnicity and social background (Smythe, 2019)

Based on this review, our contribution to the literature is twofold. First, we use a VAR framework to jointly estimate the mid-term forecasts on enrollment and public subsidies in a jurisdiction where public appropriations are tied to enrollment. We also use long-term estimates to perform a similar analysis. Second, we use the difference between long-term and mid-term forecasts to infer the impact of a pandemic on university's public subsidies. These additional findings help understanding how universities, individually or as a system, may be financially impacted by recessions. In parts based on the comparison between realized values and mid-term forecasts for the year 2021, our findings

suggest that several universities received *more* subsidies than what the effect of the recession ought to suggest.

3. Methodology

The main challenge of estimating funding and enrollment patterns is one of simultaneous endogeneity. On the one hand, an unanticipated increase in enrollment may trigger an unanticipated increase in funding. As we shall see in the next section, the structure of the funding formula makes this pretty explicit. On the other hand, an unanticipated increase in funding may *also* translate into an unanticipated increase in enrollment. Accrued funding may push universities to increase their recruitment efforts, thereby increasing enrollment. As such, the separated estimation of funding and enrollment would neglect the interconnection.

This section thus presents how we have accounted for this endogeneity problem. First, we introduce the provincial funding formula, describing how it operates. We also display the constitution of our dataset. We then present the methodology of our mid-term forecasts using VARs. We finally present how IV regressions account for the endogeneity for long-run forecasts.

3.1 The Quebec Funding Formula

The set of rules and conditions through which public appropriations are transferred to universities is dubbed a funding formula (see Frølich, 2010 for a classification). The Quebec formula is input-based, where appropriations are tied to weighted and unweighted enrollment and mostly fixed transfers (Maroy, Doray & Kabore, 2014; Bouchard St-Amant, Brabant & Germain, 2020). Roughly 85% of appropriations is tied to enrollment and 14% of the formula is, or can be approximated as, fixed transfers.

The Quebec formula is made up of five conceptual channels through which funds are evaluated for the purpose of transfer to universities: teaching activities, auxiliary services, buildings, administration and missions, and miscellaneous programs (see Table 2). Each category relies on an underlying metric to proxy for expenses. Teaching activities are measured by weighting FTEs. Each weight changes with funding families that varies with tiers and programs (see Bouchard St-Amant, Brabant & Germain, 2020, for details). Auxiliary services reflect expenses tied to campus usage and are proxied through unweighted FTEs. Combined, these two components account for more than 85% of the total funding. The buildings component reflects expenses tied to maintaining the physical infrastructure of the university. Such a category accounts for 10% of the funding and is tied to both the superficiality of the university and the price of energy. The next category, although containing different channels in principle, rely on fixed transfers. Administration services are a fixed amount, identical for each university. The funding tied to specific missions, also in the form of fixed transfer, however, varies and aims to recognize important differences in universities: geographical coverage, specialized faculties, or small graduate schools. Combined, fixed transfers amount to 3.4% of the total appropriations. The remainder of the funding formula, tied to conditions, varies with governments and with the public policy goals of the day.

Table 2. Main components of the funding formula

Category	Metric(s)	Funding (%)	Funding (M\$)	Notation
Teaching activities	Weighted FTEs	70.04	2 184	FTE_{ijt} w_j, p_1
Auxiliary services	FTEs	15.29	477	FTE_{ijt} p_2
Buildings and Offices	Squared meter, price of energy.	10.44	336	T_i
Administration and mission	Constant	3.43	107	T_i
Other	Conditional	0.45	14	ϵ_{it}

Source: author's calculations and Ministère de l'Enseignement supérieur du Québec (2021).

Three additional characteristics of the enrollment-based formula are important. First, the FTE metric determining funding is *lagged*: the funding for teaching activities in year t depends at the latest on enrollment in $t-2$. Second, funding is *smoothed*. It is tied to the highest of either the FTEs in $t-2$ or the moving average of FTEs over years $t-2$, $t-3$ and $t-4$. Such a smoothing mechanism dampens the effect of sharp declines in enrollment. Third, the highest value of enrollment is multiplied by a baseline price that converts the FTE figure in a dollar amount. There are other transitional smoothing mechanisms that are tied to some changes made to the funding weights in the year 2017. Those are explained in detail in (Bouchard St-Amant, Brabant & Germain, 2020).

In what follows, we approximate the buildings category as a fixed component. We do so because the funding category is tied to infrastructures that do not change much from one year to another, echoing the idea that capital is fixed in the short run (Frank, Bernanke, Antonovics & Heffetz, 2019). We thus denote T_i the resulting sum of the three fixed transfers to university i . By further denoting FTE_{ijt} enrollment at time t , university i and in funding family j , w_j the funding weights for each of the funding families and ϵ_{it} the remaining funding for university i at time t , then the total funding at time t (F_{it}) can be expressed by:

$$F_{it} = T_i + p_2 \max \left(\sum_j w_j FTE_{ijt-2}, \frac{1}{3} \left(\sum_j w_j FTE_{ijt-2} + \sum_j w_j FTE_{ijt-3} + \sum_j w_j FTE_{ijt-4} \right) \right) + \dots$$

$$\dots + p_1 \max \left(\sum_j FTE_{ijt-2}, \frac{1}{3} \left(\sum_j FTE_{ijt-2} + \sum_j FTE_{ijt-3} + \sum_j FTE_{ijt-4} \right) \right) + \epsilon_{it} \quad (1)$$

Equation (1) serves as a theoretical guide to develop a proper estimation model linking enrollment to funding.

3.2 Dataset

3.2.1 Enrollment and Public Funding

The dataset is a perfectly balanced panel of 18 universities over the years 1999 to 2021 (22 observations of funding and enrollment per university), for a total of 396 observations. The time range is implicitly tied to the year when enrollment-based funding was enacted in Quebec, which is year 2000 (Trahan and Carreau, 2010).

We have extracted the funding and FTEs data from the aggregated public financial statements (Ministère de l'Éducation et de l'Enseignement supérieur, 1999 to 2019) and the announced appropriations for the current year (Ministère de l'Enseignement supérieur, 2020). Because of the conditional component of the funding formula and slight changes in enrollment, the two sources differ slightly on both accounts. We kept the data from the financial statements when available and complemented with announced values for the year 2021.

These two sources are provided in portable document formats (PDFs). We developed a custom Python program to extract the data spanning over the period for each university, year and funding family, for a total of 7492 observations. This had to be complemented with human corrections. Most of the errors from the program were easily identifiable, as they related to changes in the page layout of the documents, or changes in the number of funding weights. Each author was then assigned a subset of universities to detect errors by comparing the original documents with the raw extracts. Two of the authors also re-examined the work produced by the other authors by performing the whole audit again. At this stage, the dataset has been aggregated by year, to generate its final shape and size. During the estimation, several preliminary graphics were produced, triggering automatic audits when suspicious breaks were found in enrollment patterns. The data was found to be 87% accurate after the first stage and 93% accurate at the second stage. The fact that no errors were found in the last stage, although some patterns did trigger audits, goes to suggest that the residual error in the dataset is fairly small.

3.2.2 Unemployment

Basic economic theory suggests that unemployment is a relevant proxy for the relative attractiveness of higher education. Individuals who recently lost their job may be tempted to pursue a degree in the aim of finding a new job, or simply acquire additional human capital in the interim (Dellas & Koubi, 2003). Moreover, the exercise we conduct requires forecasts on unemployment. Because the unemployment rate is forecasted by banking institutions, it is also attractive on practical grounds: the plurality of available sources.

The realized unemployment rates were gathered from two different sources. The realized unemployment rates from

1999 to 2021 were gathered from the national statistical agency (Statistics Canada, 2021). The forecasted unemployment rates stem from monthly outlooks of the main banks in Quebec (Desjardins, 2020; Arseneau & Paquet, 2020; Caranci, Burleton, Sondhi & Abdelrahman, 2020; Perrault, 2020). Two banks provide outlooks for the next four years (2021-2024), while two other banks only forecast for one year ahead (see table 3). We use the average value of those forecasts as our unemployment inputs for our mid-term models. The global outlook of unemployment in Quebec, including the forecasts, can be found in Figure 1.

Table 3. Québec Forecasting Model (Mid-Term)

F_{it}	F_{it-1}	0.481 (2.67)***
	FTE_{it-3}	0.009 (2.58)***
	Constant	-384.330 (1.33)
FTE_{it-2}	FTE_{it-3}	1.173 (6.19)***
	FTE_{it-4}	-0.542 (2.19)**
	pop_{it-4}	0.033 (2.81)***
	u_{it-3}	3345.207 (2.24)**
	Constant	-209 629.960 (3.17)***
N		20

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Source: Authors' classification and Québec Department of Higher Education.



Figure 1. Quebec unemployment rate

3.2.3 Demography

Long-term trends in enrollment are best represented by demographic patterns. They have the advantage of being reliably forecasted over long periods, allowing us to support our forecasting analysis. As such, we used the yearly total population size, the yearly 18 to 25 years old population size, as well as the number of yearly immigrants in Quebec. Each of these components capture segments of the population that may reflect a fundamental basin for enrollment.

Our demographic data comes from both the national and the provincial statistical agencies. In a similar fashion to unemployment, existing demographic values were extracted from Statistics Canada (2020). This source is combined with forecasted demographic trends over the next 20 years, providing us with support for our long-term forecasts (Institut de la statistique du Québec, 2019). The resulting population trends are presented in Figures 2-a to 2-c.

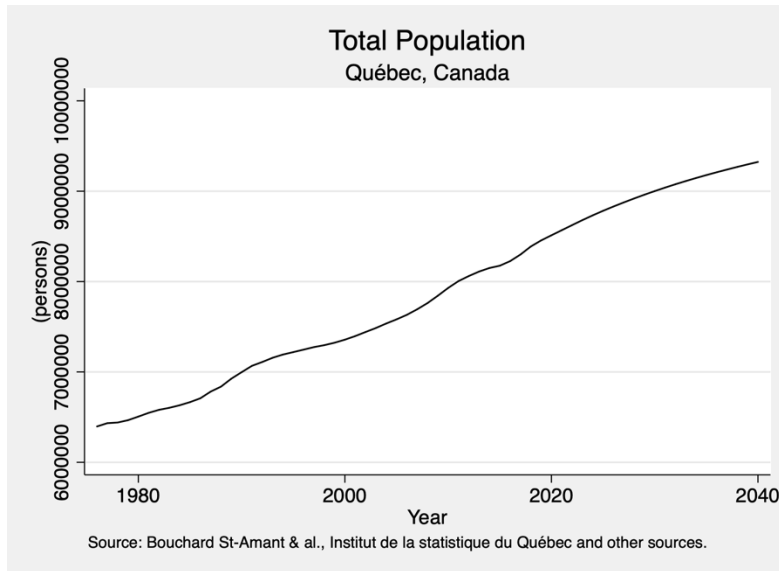


Figure 2a. Total population trend and projection

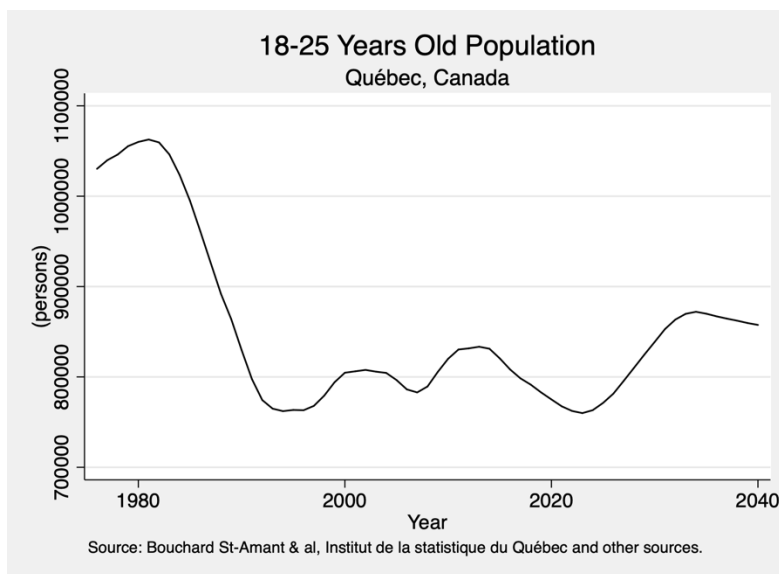


Figure 2b. 18-25 years old population trend and projection

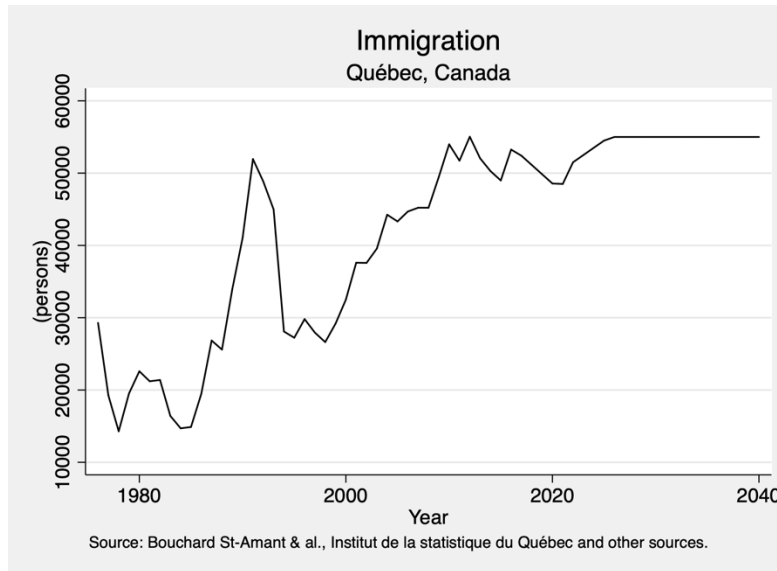


Figure 2c. Immigration trend and forecast

3.4 Vector Autoregressive Models

Following the notation used to develop equation (1), denote FTE_{it} (without the index j) the *total unweighted* enrollment in university i at time t . We then show in Appendix B that a linear approximation of equation (1) around the values of FTE_{it-2} , FTE_{it-3} and FTE_{it-4} , yields the following reduced form:

$$F_{it} = \beta_0 + \beta_1 FTE_{it-2} + \beta_2 FTE_{it-3} + \beta_3 FTE_{it-4} + \beta_4 F_{it-1} + \beta_5 F_{it-2} + \varepsilon_{it}, \quad (2)$$

where the coefficients β_1 to β_3 capture the average effect of total enrollment at a given lag in time, β_0 denotes fixed transfers, and ε_{it} denotes the global estimation error induced by both the linear approximation and the conditional funding rules in the original formula. If it were not of lagged values in the funding variables, the error term ε_{it} would be correlated with past funding values, as a consequence of our linear approximation. The smoothing mechanism described in (1) is indeed aimed at mitigating changes in funding over time (and eliminated by linearization). And so, when statistically relevant, lagged trends on the funding variable are added as explanatory variables.

The estimation of enrollment is a straightforward connection between demographic patterns and short-term deviations proxied by unemployment. Denoting pop_t , $pop18_t$, im_t and u_t for respectively the general population count, the 18 to 25 years old population count, the immigration count and the unemployment rate, our generic reduced form is given by:

$$FTE_{it} = \alpha_0 + \alpha_1 pop_{t-L1} + \alpha_2 pop18_{t-L2} + \alpha_3 im_{t-L3} + \alpha_4 u_{t-L4} + \alpha_5 FTE_{it-1} + \alpha_6 FTE_{it-2} + \varepsilon'_{it}, \quad (3)$$

where the lag selection ($L1 \dots L4$) over variables is chosen so as to maximize the explanatory power of each variable. For population variables, the number of lags means that the current population, older by the specified number of lags, better captures the enrollment trends. The lags on unemployment, however, is an indication of the speed at which individuals adapt their behaviour on the labour market. For instance, an absence of lag would mean that the current unemployment drives the current enrollment, while one lag would mean that it takes (roughly) one year for individuals to move out of the labour market and in a university.

In a similar fashion to funding, we add a lagged variable on enrolment itself when justified by a rejection of the null hypothesis on the associated coefficient. The interpretation is rooted in the fact that it takes more than a year to complete most degrees. Accordingly, past enrollment can help predict current enrollment.

Equations (2) and (3) form a typical restricted VAR, an estimation technique that accounts for the interdependence in error terms. We first perform the statistical analysis with an unrestricted estimation, and then reduce the model only to variables where the null hypothesis is rejected on the associated coefficient. By doing so, we reduce the forecast

margins by removing noisy variables with a weak explanatory power. We then use the resulting model to perform forecasts over the next four years, with bootstrapped margins of errors over 400 replications (Davison & Hinkley, 1997).

3.5 Instrumental Variables

The estimation framework for long-term forecasts is much simpler. The unemployment variable is dropped, and there is no multiplicity in time-lags. We thus focus on the core relationship that exists between enrollment and funding:

$$F_{it} = \beta_0 + \beta_1 FTE_{it-L}^{max} + \varepsilon_{it}, \quad (4)$$

$$FTE_{it-L}^{max} = \alpha_0 + \alpha_1 pop_{t-L1} + \alpha_2 pop18_{t-L2} + \alpha_3 im_{t-L3} + \varepsilon'_{it}, \quad (5)$$

where the lags $L, L1, \dots, L3$ are chosen empirically to best reproduce the long-term patterns and FTE_{it-L}^{max} is a variable derived from the core component of the funding formula (1):

$$FTE_{it-L}^{max} \equiv \max \left(\sum_j FTE_{ijt-2}, \frac{1}{3} \left(\sum_j FTE_{ijt-2} + \sum_j FTE_{ijt-3} + \sum_j FTE_{ijt-4} \right) \right). \quad (6)$$

Because none of the long-term drivers of enrollment can be reasonably explained by unexpected changes in the funding to universities, the endogeneity matters only when shocks in enrollment occur (and affect funding). Accordingly, a two-stage least square IV regression is used to estimate (4) from (5) (Greene, 2018). Similar in spirit to our VAR methodology, variables with a coefficient that fail to reject the null hypothesis are removed to reduce forecasting margins of error.

4. Results

We first present our forecast for the system as a whole. By the simple virtue of aggregation, random errors occurring at the university level are greatly reduced, therefore providing more accurate forecasts. We then present three additional forecasts for selected universities. Finally, we come back to the system analysis to evaluate differences between the short and the long run. Appendix A also presents summary results for the 14 other universities.

4.1 Provincial Forecasts

4.1.1 Mid-term

We present the mid-term forecasting model in Table 3 and the graphical presentation of the forecast in Figure 3. In both cases, panel *a* presents the forecast on the public subsidy (in millions) while panel *b* presents the evolution of full-time equivalent enrollment (in thousands). Table 4, panel a, reveals that the best predicting equation for the current period's funding includes 48% of past year's funding and 9,000\$ per FTEs (lagged by three years). Other variables have coefficients that are statistically indiscernible from zero. Panel b shows coefficients of 1.17 for FTEs with a two-year lag and of -0.54 of enrollment for FTEs with a three-year lag. This reduction reflects the combination of graduation rates and drop-outs, and suggests a one-year surge in enrollment, followed by a reduction in enrollment two years later. The population coefficient suggests that roughly three percent of the population enters the university system yearly, although the pattern between enrollment and the population count is lagged by two years. Finally, unemployment has a positive effect with roughly 3,300 additional FTEs for a 100 basis points increase in unemployment.

The graphical representation of forecasts suggests a slight decrease in funding, assuming of course no significant changes to the funding formula. This decrease in funding is induced by decreases in enrollment, partly induced by the post-recession unemployment rate returning to normal values in the upcoming years. This forecasted decline in funding however has a margin of error large enough to allow for increases. Because enrollment patterns are forecasted to increase beyond year 2023, the funding is forecasted to increase again in 2025.

Table 3. Québec forecasting model (Mid-term)

F_{it}	F_{it-1}	0.481 (2.67)***
	FTE_{it-3}	0.009 (2.58)***
	Constant	-384.330 (1.33)
FTE_{it-2}	FTE_{it-3}	1.173 (6.19)***
	FTE_{it-4}	-0.542 (2.19)**
	pop_{it-4}	0.033 (2.81)***
	u_{it-3}	3345.207 (2.24)**
	Constant	-209 629.960 (3.17)***
N		20

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Source: Authors' calculations and Québec Department of Higher Education.

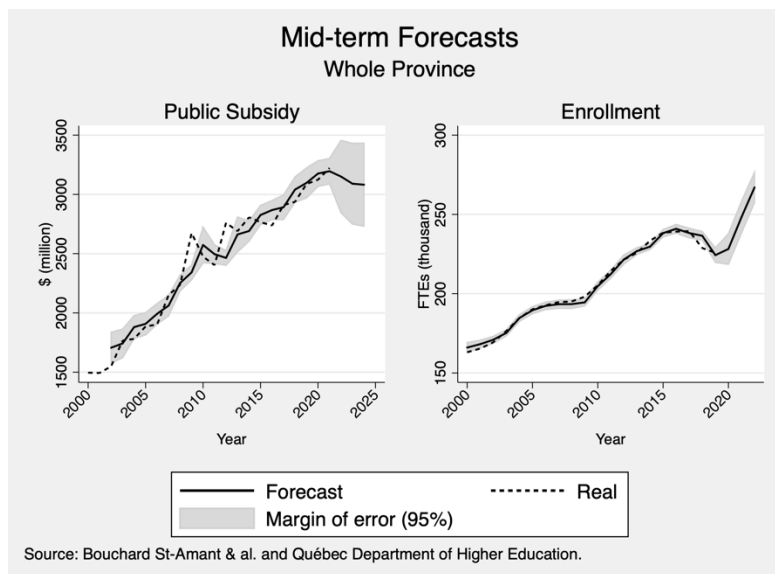


Figure 3. Québec Forecast (Mid-term)

4.1.2 Long-term

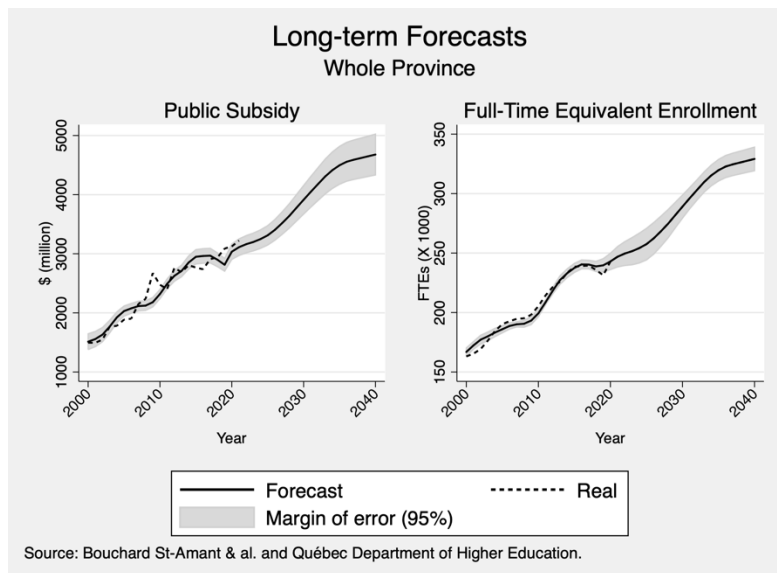
Long-term provincial forecasts are presented in Table 4 and Figure 4. Because mid-term dynamics are not modeled, deviations from the forecasts are more frequent (and thus margins of error larger). In particular, mid-term forecasts for funding predict a milder decline than what long-term forecasts present because changes in enrollment patterns do not account for unemployment, nor the autocorrelation with past funding. However, forecasts go as far as 2040 and suggest a steady increase both in terms of funding and enrollment.

Table 4. Québec Forecasting Model (Long-Term)

	F_{it}
FTE_{it}	19.091 (18.60)***
Constant	-1603.150 (7.94)***
R^2	0.92
N	22

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Source: Authors' calculations and Québec Department of Higher Education.



Figures 4. Québec Forecast (Long-term)

4.2 Three Selected Universities

Not all universities are similar in terms of mid-term trends. Some seem unaffected by shocks to unemployment, while others seem to rely more on past enrollment values than provincial demographics. Globally, 8 universities respond to unemployment, as well as the Université du Québec system as a whole, and the whole provincial system as well. In broad terms, large universities respond to unemployment as well as a few regional universities. To outline these differences, we present the mid-term forecasts for three separate universities: one large university (Université de Sherbrooke), one small regional university (Université du Québec en Abitibi Témiscamingue), and one English speaking university (Concordia University).

The peculiarity of Université de Sherbrooke's forecast is the regularity of its enrollment pattern. This is reflected by the

fact that unemployment has no impact on enrollment. Because of that regularity, the simplest model for this university turns out to be that past enrollment predicts future enrollment. This university has a distinctive trait from other universities that may very well explain why unemployment is not a significant driver: it relies on an alternance between internship semesters and classes semesters, where students are registered all year long and alternate between mandatory internships and more traditional courses (Université de Sherbrooke, 2020). This tighter, year-round curriculum may make it less appealing for recently unemployed individuals, thereby shielding Université de Sherbrooke from short-term fluctuations.

Université du Québec en Abitibi-Témiscamingue is also unaffected by unemployment but is driven by the size of the total population. As such, it shows a fairly straightforward increase in enrollment and in funding over time.

Table 5. Université de Sherbrooke’s Forecasting Model (Mid-Term)

F_{it}	FTE_{it-3}	39.103 (4.64)***
	FTE_{it-4}	-20.005 (2.34)*
	Constant	-95,713 (5.61)***
FTE_{it-2}	pop_{it-4}	0.008 (15.34)***
	pop_{it-3}	229.405 (1.13)
	Constant	-52,309 (9.04)
N		20

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Source: Authors’ calculations and Québec Department of Higher Education.

Table 6. Université du Québec en Abitibi-Témiscamingue Forecasting Model (Mid-Term)

F_{it}	FTE_{it-3}	27.345 (14.77)***
	Constant	-10 485.640 (3.56)***
FTE_{it-2}	FTE_{it-3}	1.100 (7.28)***
	FTE_{it-4}	-0.390 (2.70)***
	pop_{it-4}	0.00037 (3.44)***
	Constant	-2494.522 (3.70)***
N		19

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Source: Authors’ calculations and Québec Department of Higher Education.

Table 7. Concordia University Forecasting Model (Mid-Term)

F_{it}	FTE_{it-4}	7.835 (3.22)***
	Constant	2005.135 (0.07)
FTE_{it-2}	FTE_{it-3}	0.605 (3.16)***
	FTE_{it-4}	0.400 (2.01)**
	u_{it}	916.768 (4.10)***
	Constant	-6731.945 (2.55)**
N		20

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Source: Authors' calculations and Québec Department of Higher Education.

Concordia university, on the other hand, is sensitive to changes in unemployment, but is not statistically tied to the demographics of the province. The university's pool of students is 30% from outside the province (Concordia University, 2020). Moreover, 32% of the student body has another language than English or French as their mother tongue (id.). Concordia thus has a larger pool for recruitment than what the provincial demographics can model. Combined with the limited 22 observations per university, this translates into past enrollment trends giving better forecasts than provincial demographics.

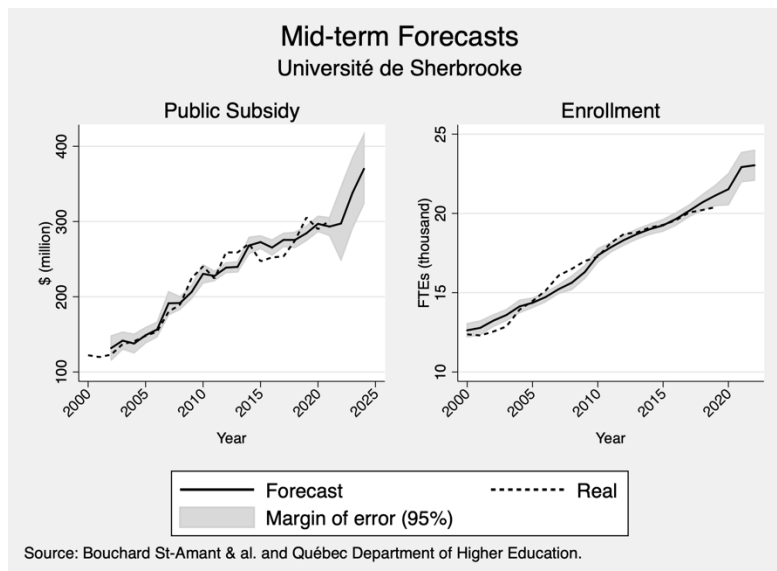


Figure 5. Université de Sherbrooke Forecast (Mid-term)

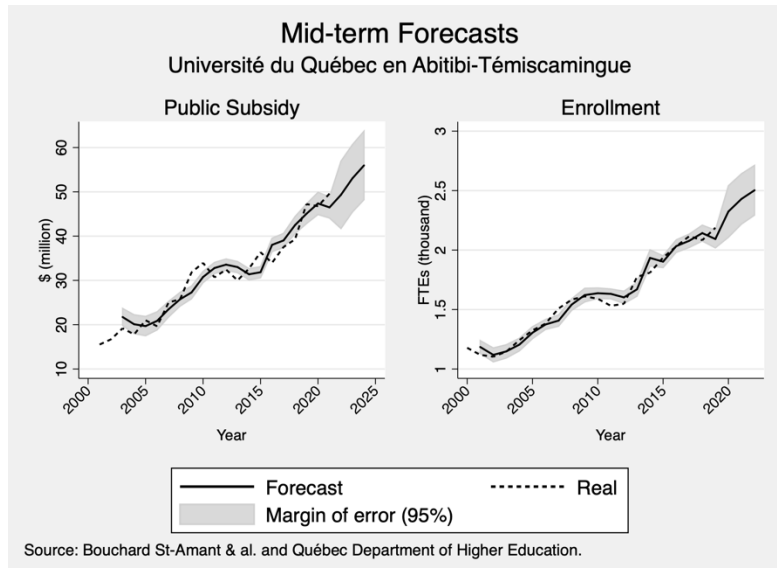


Figure 6. Université du Québec en Abitibi-Témiscamingue Forecast (Mid-term)

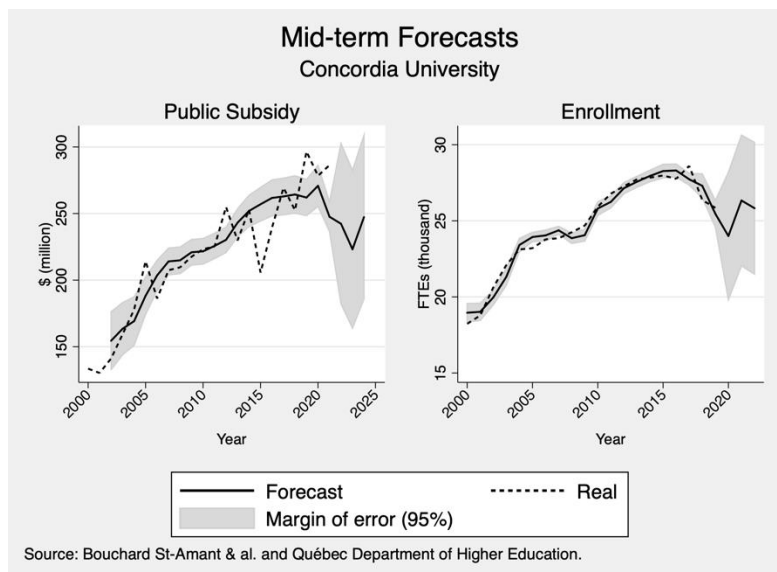


Figure 7. Concordia University Forecast (Mid-term)

In terms of forecasting models, two broad results stem from our analyses. First, smaller universities have proportionally larger variations in enrollment patterns, thus yielding proportionally larger predictive margins of error. We interpret this as an organizational incarnation of the law of large numbers: because smaller universities have fewer programs through which they can compensate for local enrollment shocks, aggregate variations turn out to be larger in proportion. Second, English speaking universities have patterns of their own, less tied to the provincial population patterns. We interpret this last fact as an indication that English speaking universities have a greater proportion of enrollment coming from an international pool, and that incidentally, the demographics of the province have a weaker predictive value.

4.3 Impact of COVID on Universities' Finances

Our modelling approach allows us to distinguish long-term trends from mid-term trends. In this section, we thus use them as a counterfactual experiment to isolate their respective impacts on funding. Prior to year 2022, we can further decompose the trends by analyzing the difference between the realized values for funding and the mid-term trend. If one denotes FL_{it} , FM_{it} and F_{it} , as respectively the long-term forecast (eq. 4), the mid-term forecast (eq. 2) and the realized value, the following identity is a useful decomposition to interpret the source of the realized funding values:

$$F_{it} = FL_{it} + (FM_{it} - FL_{it}) + (F_{it} - FM_{it}) \tag{6}$$

In (6), the first term of the right-hand side represents the long-term funding (eq. 4), the second term represents the known impacts of the mid-term deviations (eq. 4 - eq. 2) and the last term represents the unanticipated changes in funding (ϵ_{it} in eq. 2). In what comes below, we interpret the last term as the additional funding awarded to universities during the pandemic.

The first and last terms of (6) can be computed with a margin of error, as both values stem directly from an estimated equation. If the middle term is conceptually easier to understand as the difference between the two forecasting methods, it does not have a margin of error, as it comes from the difference of two equations estimated separately. Another way to estimate the middle term is to compute the impact of unemployment on funding through the combination of equations (2) and (3), with the associated margin of error:

$$\Delta F_{it} = \beta_1 \alpha_4 u_{t-L4} + \beta_2 \alpha_4 u_{t-1-L4} + \beta_3 \alpha_4 u_{t-2-L4} \tag{7}$$

This second approach (7) yields an estimate different than the first approach, with the benefits of having margins of errors. Since both approaches are instructive, we report them in Table 8, as well as the rest of the decomposition, for year 2021.

Table 8. Short to Mid Term Decomposition for Year 2021

	$FL - FM$	ΔF	ϵ
Qc	84.45	159.67*	26.56
		(-31.33, 350.67)	(-107.79, 107.79)
Bishop	-4.4214	-0.2067	2.7333*
		(-1.2880, 0.8746)	(-2.867, 2.867)
Concordia	1.4357	53.724**	38.748***
		(25.549, 81.899)	(-11.122, 11.122)
HEC	-6.930	-6.9752	-0.3960
		(-43.914, 29.964)	(-3.8121, 3.8121)
McGill	-14.875	21.546	30.183**
		(-17.935, 61.026)	(-27.788, 27.788)
Poly	-3.317	3.0334	3.8491
		(-2.2145, 8.2815)	(-4.5548, 4.5548)
Montr éal	-4.6523	28.204*	18.999**
		(-3.3093, 59.716)	(18.504, 18.504)
Sherbrooke	5.2230	49.337	8.2397
		(-38.675, 137.35)	(-11.830, 11.830)
Laval	-2.9501	10.971	8.3394
		(-4.8713, 26.814)	(-15.701, 15.701)
UQ System	13.671	76.490***	30.095**
		(2.1633, 150.82)	(-20.121, 20.121)
ENAP	n/a	n/a	6.8056
			(-1.7444, 1.7444)

Table 8 continued from previous page

ETS	4.2089	0.00 n/a	-7.8247 (-9.1636, 9.1636)
INRS	-7.232	-1.597 (-5.1774, 1.9844)	1.2333 (-2.357, 2.357)
UQAC	0.2931	8.4478** (0.3211, 16.574)	0.8413 (-3.8149, 3.8149)
UQAM	-13.934	28.113 (-13.173, 63.398)	32.197*** (-10.294, 10.294)
UQAR	1.3422	7.9022** (0.3809, 15.424)	10.326*** (-3.068, 3.068)
UQAT	-1.578	2.503 (-2.223, 7.2281)	3.1094** (-2.4041, 2.4041)
UQO	0.1995	4.9727 (-6.7581, 16.704)	2.7835 (-3.6514, 3.6514)
UQTR	4.8770	25.617*** (8.1030, 43.131)	3.0454 (-5.1796, 5.1796)
TELUQ	2.5882	-4.6512 (-15.051, 5.7491)	-5.6147** (-5.3349, 5.3349)

Source: Authors' calculations. Legend: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$. The 95% confidence interval is in parentheses below the forecast. The not applicable mention (n/a) means that the data is not available. Because equations are estimated separately, the sum of funding (or funding variations) does not equal the total funding (or total variations).

Table 8 shows that 7 universities, or systems of universities, see their finances increase because of the unemployment rate (column ΔF). For instance, the unemployment rate increased the total funding by \$ 159.67 millions for the provincial system as a whole. The Universit  du Qu bec system got an estimated additional \$ 76.49 millions for the same reason. This goes to show the concrete effect of unemployment on enrollment which, in turn, increases the public subsidy through the funding formula. Two metropolitan universities, Universit  de Montreal and Concordia University, seem to have benefited from additional subsidies through the effect of unemployment. The two other universities that see statistically significant increases in funding are regional universities of the Universit  du Qu bec system.

All of the remaining universities do not have a statistically significant variation in funding induced by unemployment. This can be so for several reasons. First, some of those universities' enrollment patterns are unaffected by unemployment. Second, for 3 estimated models, the non-linear combination of the unemployment effect on funding (eq. 7) is not statistically different from zero, although unemployment has a statistically significant effect on enrollment. This can be the case if the smoothing mechanisms on funding -- in our estimates, the lagged variables on funding -- has a greater effect on funding than unemployment itself. Third, there may simply not be enough statistical power, with 22 observations, to distinguish the result from the null effect. This last case is a plausible explanation for smaller universities.

In terms of outliers during year 2021, eight universities seem to have benefited from the pandemic beyond what the unemployment rate can explain: Bishop's University,  cole nationale d'administration publique, McGill University, Universit  de Montr al, Universit  du Qu bec   Montr al, Universit  du Qu bec   Rimouski and Universit  du Qu bec en Abitibi-T niscamingue. Since four of those universities are in the Universit  du Qu bec system, including the largest one (UQAM), the statistically significant effect of additional funding unexplained by unemployment also shows up in the aggregate, with an extra \$ 30.10 millions for the system. On the other end of the spectrum, one university stands out as having current subsidies below its predicted value during the pandemic, which is Universit  T LUQ. This is so because enrollment went much higher two years ago than what its normal trend predicts, pushing

funding upwards. Thus, predicted funding is higher than what is actually being given.

5. Conclusion

Our paper presents several insights that help understanding how university systems respond to economic conditions which can be helpful for practitioners and researchers alike. First, we develop a methodology to estimate simultaneously enrollment and public subsidies. In jurisdictions where appropriations depend on enrollment, this is a particularly apt framework to model the evolution of public finances and its interdependence with enrollment. Second, it is shown that unemployment drives enrollment upwards in some universities or university systems, including the whole system of universities. This means that during (or around) recessions, public subsidies are forecasted to *increase* if the funding policies are left as is. Moreover, we further find that during the current pandemic, inducing a recession, the increase in funding went beyond what changes in unemployment could explain. Thus, some factors left out of our analysis increase funding by more than what it ought to. One interpretation is one of *overshooting*, where public authorities gave more funding than what would be the typical value during a recession. In all cases, forecasting models such as the ones developed in this paper can help strengthen our understanding of how university systems respond during recessions, and thus, facilitate planning.

On top of these findings, we also find qualitative forecasting results of interest. First, the margin of error of the forecast is generally larger for smaller universities, suggesting that they are more prone to enrollment shifts from one program to another, a phenomenon that vanishes for large universities. This goes to suggest that the lack of effect of the unemployment variable for some of these universities is simply a consequence of a lack of statistical power. Second, English speaking universities are better explained by autocorrelation patterns than by the French speaking driven demographics of the province. In broader terms, universities that depend more on out-of-jurisdiction enrollment, rather than within-jurisdiction enrollment, are better explained by autocorrelation patterns, or additional variables not found in this paper.

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Appendix A

Remaining Universities' Forecasts

All universities have both a long-term and a mid-term model, with the exception of École nationale d'administration publique (ENAP), whose enrollment pattern cannot be characterized in the long run. All universities have their estimated mid-term model presented below, as well as tables for both short-term and long-term forecasts for selected numerical values. Each university has also two graphical representations of forecasts. Only a subset of the forecasts are shown. Mid-term forecasts are available from 2018-2019 to 2023-2024 and long-term forecasts are available from 2020-2021 to 2039-2040. However, because some universities use lagged variables of population or immigration, the last forecasted year can be 2037-2038 or 2038-2039. All FTE figures are in thousands and all Subsidies figures are in millions of dollars. Consequently, all estimated coefficients in all equations reflect those units.

A.1 Bishop's University

Table A1. Bishop's University Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	2.0682	1.9406	1.8292	1.7416
	Forecast	2.2011	2.1390	2.0355	1.9463
	95% Lower Bound	2.3340	2.3375	2.2417	2.1511
Subsidy	95% Upper Bound	28.829	30.811	30.615	36.518
	Forecast	26.766	28.332	27.748	28.967
	95% Lower Bound	24.703	25.852	24.881	21.416

Source: Authors' calculations and Québec Department of Higher Education.

Table A2. Bishop's University Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	2.2377	2.5180	2.8605
	Forecast	2.1433	2.4268	2.6639
	95% Lower Bound	2.0491	2.3357	2.4673
Subsidy	95% Upper Bound	36.693	38.644	39.691
	Forecast	32.200	35.405	36.815
	95% Lower Bound	27.706	32.166	33.938

Source: Authors' calculations and Québec Department of Higher Education.

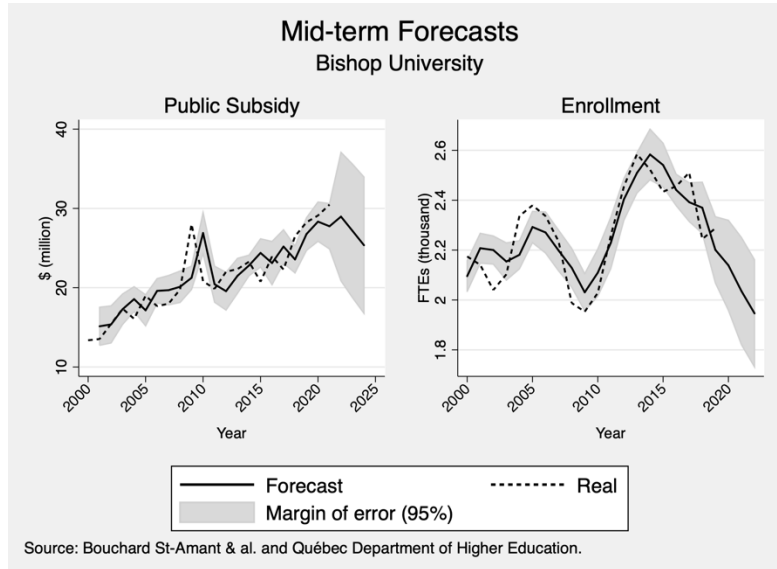


Figure A1. Bishop’s University Forecast (Mid-term)

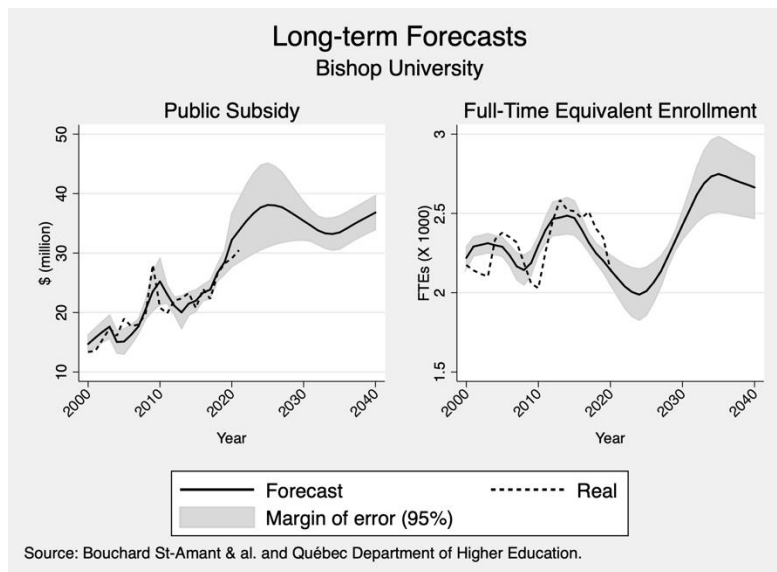


Figure A2. Bishop’s University Forecast (Long-term)

$$F_t = -4.0771 + 0.7417 F_{t-1} + 0.0047 FTE_{t-3} + \varepsilon_t, \tag{A1}$$

$$FTE_{t-2} = -3315.26 + 0.4991 FTE_{t-3} + 0.0056 pop18_{t-5} + \varepsilon_t. \tag{A2}$$

A.2 Concordia University

See the main text for the short-term forecasting model and projections.

Table A3. Concordia University Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	27.689	30.927	31.916
	Forecast	28.537	32.483	34.090
	95% Lower Bound	26.840	29.371	29.742
Subsidy	95% Upper Bound	263.29	311.04	325.69
	Forecast	248.36	289.32	301.83
	95% Lower Bound	233.43	267.60	277.96

Source: Authors' calculations and Québec Department of Higher Education.

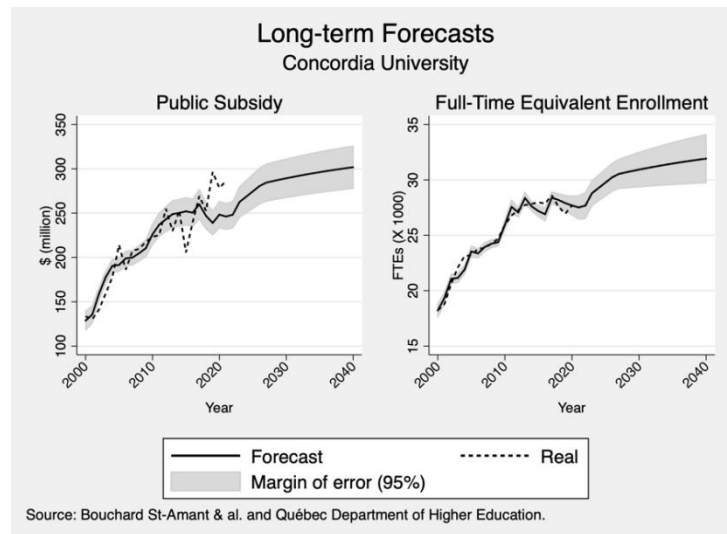


Figure A3. Concordia University Forecast (Long-term)

$$F_t = -25.021 + 0.1034 FTE_{t-3} + \varepsilon_t, \tag{A3}$$

$$FTE_{t-2} = -7235.19 + 0.6889 FTE_{t-3} + 0.3264 FTE_{t-4} + 944.57 u_{t-3} + \varepsilon_t. \tag{A4}$$

A.3 École nationale d'administration publique

Table A4. École nationale d'administration publique Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	0.6770	0.7154	0.7843	0.9069
	Forecast	0.6260	0.6156	0.6046	0.5977
	95% Lower Bound	0.5751	0.5157	0.4249	0.2884
Subsidy	95% Upper Bound	14.349	13.748	13.366	13.075
	Forecast	12.755	11.621	11.621	11.278
	95% Lower Bound	11.161	10.385	9.8768	9.4812

Source: Authors' calculations and Québec Department of Higher Education.

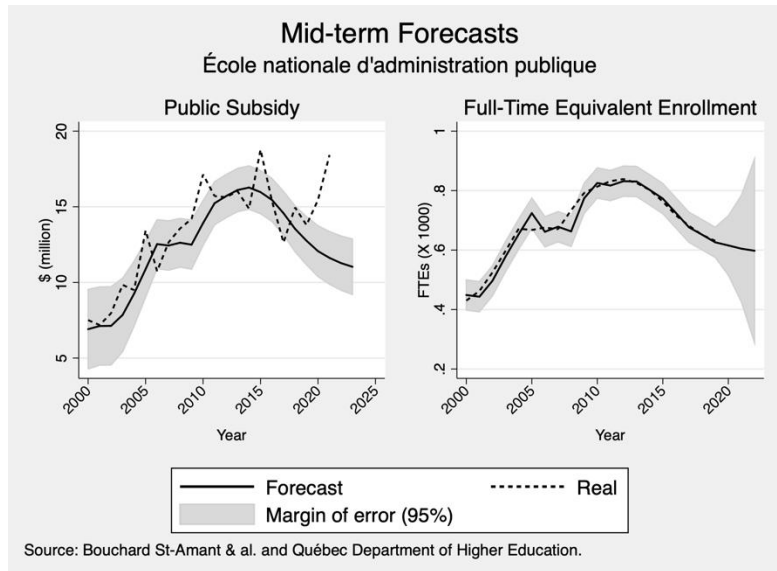


Figure A4. École nationale d'administration publique Forecast (Mid-term)

$$FTE_t^{max} = 0.6239 + 1.6836 FTE_{t-1}^{max} - 0.7483 FTE_{t-2}^{max} + \varepsilon_t. \tag{A5}$$

A.4 École de technologie supérieure

Table A5. École de technologie supérieure Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	8.5104	9.0613	9.3822	9.7814
	Forecast	8.1150	8.1874	8.4451	8.8282
	95% Lower Bound	8.0638	7.3135	7.5081	7.8749
Subsidy	95% Upper Bound	132.21	131.14	133.44	145.42
	Forecast	123.20	122.27	116.46	122.92
	95% Lower Bound	114.19	113.39	115.12	100.41

Source: Authors' calculations and Québec Department of Higher Education.

Table A6. École de technologie supérieure Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	8.9192	13.107	15.981
	Forecast	8.4578	12.529	15.227
	95% Lower Bound	7.9963	11.951	14.473
Subsidy	95% Upper Bound	128.03	186.72	239.73
	Forecast	121.25	174.88	223.15
	95% Lower Bound	114.46	163.04	206.57

Source: Authors' calculations and Québec Department of Higher Education.

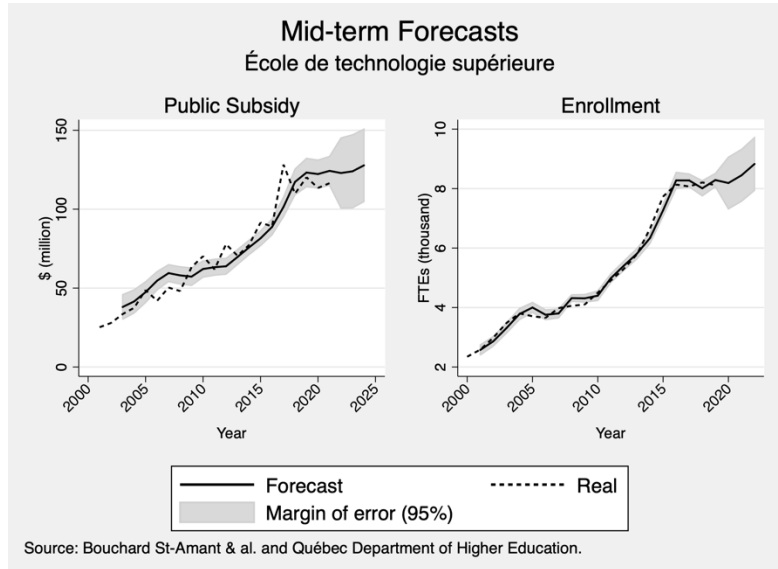


Figure A5. École de technologie supérieure Forecast (Mid-term)

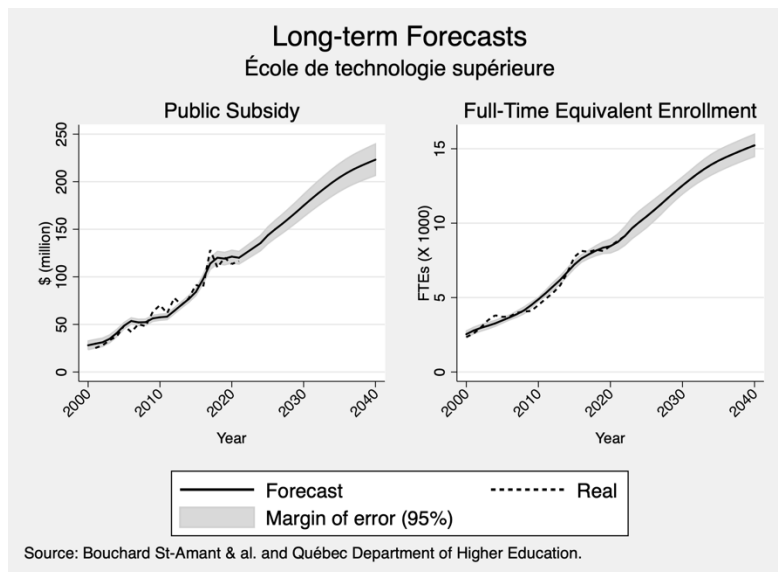


Figure A6. École de technologie supérieure Forecast (Long-term)

$$F_t = 3517.31 + 14.716 FTE_{t-3} + \varepsilon_t, \tag{A6}$$

$$FTE_{t-2} = -8663.60 + 1.2992 FTE_{t-3} - 0.5320 FTE_{t-4} + 0.0013 pop_{t-7} + \varepsilon_t. \tag{A7}$$

A.5 Haute Études Commerciales

Table A7. Haute Études Commerciales Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	9.2078	9.3523	9.1493	9.2180
	Forecast	8.8807	8.7758	8.5753	8.6229
	95% Lower Bound	8.3887	8.1992	8.0014	8.0278
Subsidy	95% Upper Bound	84.423	88.723	86.911	92.943
	Forecast	80.230	84.552	82.702	77.923
	95% Lower Bound	76.037	80.381	79.286	62.903

Source: Authors' calculations and Québec Department of Higher Education.

Table A8. Haute Études Commerciales Forecasting Results (Long-Term)

		2018-2019	2028-2029	2037-2038
FTEs	95% Upper Bound	9.0824	8.1260	8.2082
	Forecast	8.8444	7.8051	7.9137
	95% Lower Bound	8.6064	7.4842	7.6192
Subsidy	95% Upper Bound	87.096	70.528	72.235
	Forecast	83.670	68.073	69.703
	95% Lower Bound	80.244	65.618	67.173

Source: Authors' calculations and Québec Department of Higher Education. The year 2038 is used as it is the last available forecast.

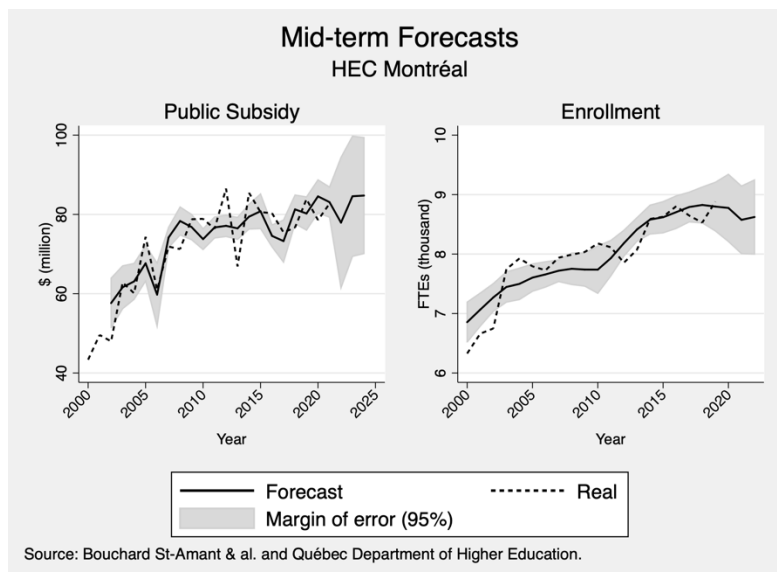


Figure A7. Haute Études Commerciales Forecast (Mid-term)

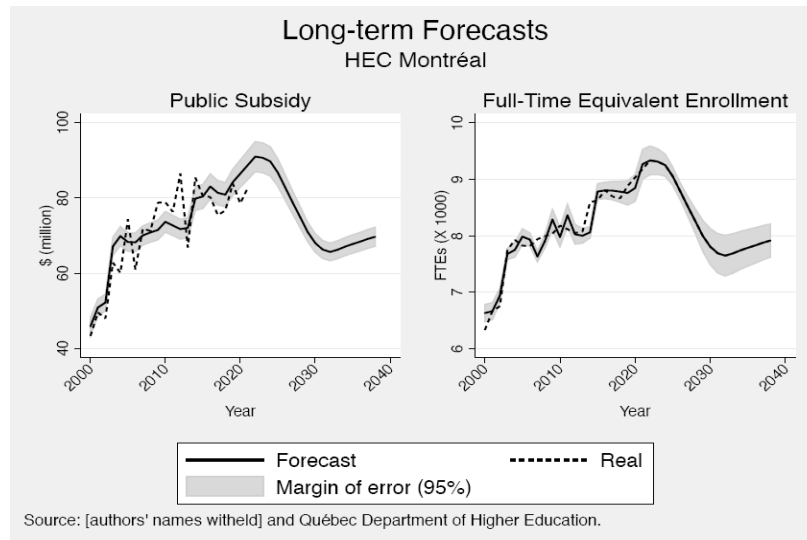


Figure A8. Haute Études Commerciales Forecast (Long-term)

$$F_t = 23.110 - 0.0093 FTE_{t-3} + 0.0161 FTE_{t-4} + \varepsilon_t, \tag{A8}$$

$$FTE_{t-2} = -10069.75 - 47.455 u_{t-3} + 0.0111 pop18_{t-3} + 0.00120 pop_{t-2} + \varepsilon_t. \tag{A9}$$

The coefficient associated with FTE_{t-3} is not statistically different from zero.

A.6 Institut National de Recherche Scientifique

Table A9. Institut National de Recherche Scientifique Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	0.4808	0.5049	0.4888	0.4947
	Forecast	0.4582	0.4696	0.4522	0.4613
	95% Lower Bound	0.4354	0.4343	0.4156	0.4278
Subsidy	95% Upper Bound	53.828	59.795	60.068	65.036
	Forecast	52.105	57.308	58.955	58.899
	95% Lower Bound	50.383	54.822	55.353	52.762

Source: Authors' calculations and Québec Department of Higher Education.

Table A10. Institut National de Recherche Scientifique Forecasting Results (Long-Term)

		2018-2019	2028-2029	2037-2038
FTEs	95% Upper Bound	471.60	490.66	635.35
	Forecast	450.85	525.82	607.23
	95% Lower Bound	430.09	560.97	579.10
Subsidy	95% Upper Bound	29240.93	36898.37	40440.84
	Forecast	27528.73	34064.98	36007.07
	95% Lower Bound	25816.52	31231.59	31573.31

Source: Authors' calculations and Québec Department of Higher Education. Year 2038 is used as the last forecast since the variable choice prevents 2039 to be forecasted.

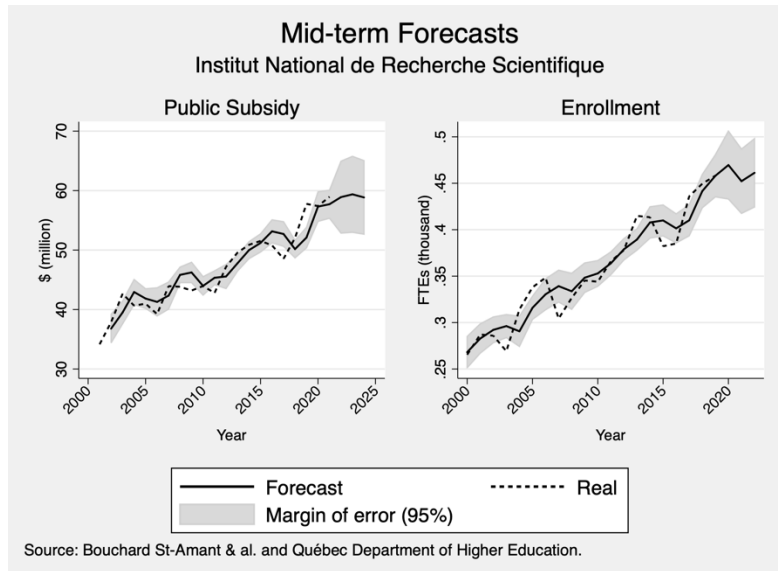


Figure A9. Institut National de Recherche Scientifique Forecast (Mid-term)

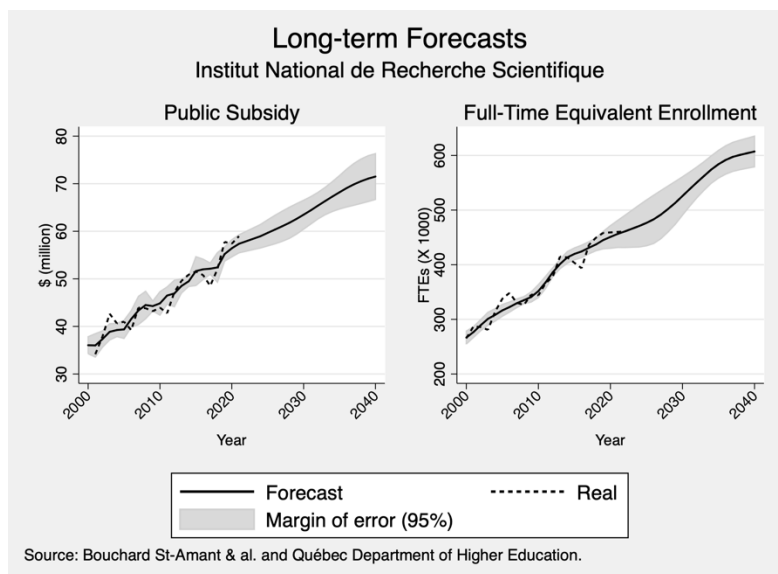


Figure A10. Institut National de Recherche Scientifique Forecast (Long-term)

$$F_t = 8217.29 + 0.5185 F_{t-1} + \varepsilon_t, \tag{A10}$$

$$FTE_{t-2} = -387.85 + 0.0001 Pop_{t-5} + \varepsilon_t. \tag{A11}$$

A.7 McGill University

Table A11. McGill University Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	27.260	34.537	36.551	37.265
	Forecast	26.405	30.816	32.848	33.698
	95% Lower Bound	22.926	27.094	29.146	30.131
Subsidy	95% Upper Bound	372.03	384.87	373.31	435.74
	Forecast	354.96	367.68	375.70	357.42
	95% Lower Bound	337.94	350.47	317.73	279.09

Source: Authors' calculations and Québec Department of Higher Education.

Table A12. McGill University Forecasting Results (Long-Term)

		2018-2019	2028-2029	2037-2038
FTEs	95% Upper Bound	32.477	43.060	46.820
	Forecast	30.852	40.552	43.823
	95% Lower Bound	29.228	30.06	40.83
Subsidy	95% Upper Bound	373.13	585.07	656.83
	Forecast	350.64	526.72	506.11
	95% Lower Bound	328.14	468.37	515.38

Source: Authors' calculations and Québec Department of Higher Education. Year 2038 is used as the last forecast since the variable choice prevents 2039 to be forecasted.

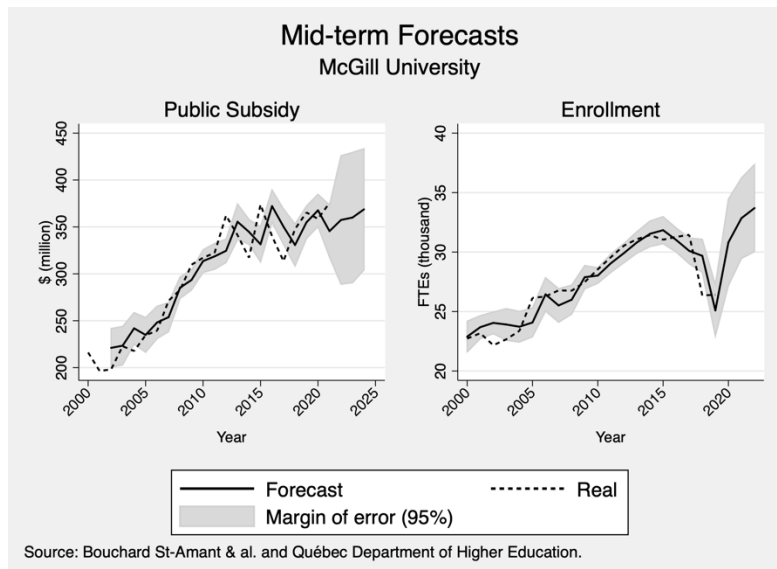


Figure A11. McGill University Forecast (Mid-term)

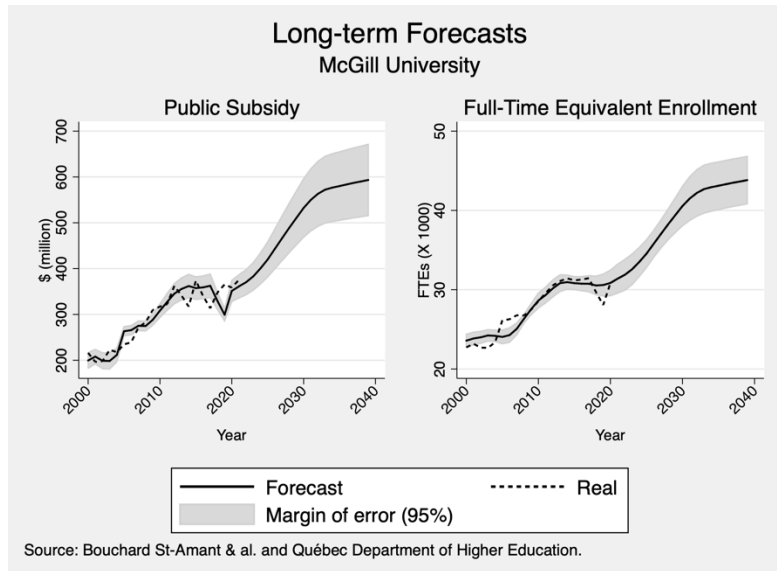


Figure A12. McGill University Forecast (Long-term)

$$F_t = 10.194 + 0.6838 F_{t-1} + 0.0034 FTE_{t-3} + \varepsilon_t, \tag{A12}$$

$$FTE_{t-2} = -31891.64 + 0.8532 FTE_{t-3} + 0.001 Pop_{t-6} + 1235.16 u_t + \varepsilon_t. \tag{A13}$$

A.8 Polytechnique Montréal

Table A13. Polytechnique Montréal Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	7.7232	8.0969	8.3245	8.5074
	Forecast	7.1978	7.5390	7.7979	7.9538
	95% Lower Bound	6.6724	6.9810	7.2712	7.4003
Subsidy	95% Upper Bound	111.12	105.54	125.57	129.33
	Forecast	106.00	104.84	106.49	110.26
	95% Lower Bound	100.88	96.433	87.409	91.199

Source: Authors' calculations and Québec Department of Higher Education.

Table A14. Polytechnique Montréal Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	7.3449	10.167	12.048
	Forecast	7.0897	9.4814	11.044
	95% Lower Bound	6.8345	8.7962	10.040
Subsidy	95% Upper Bound	104.50	156.79	188.77
	Forecast	101.68	149.65	178.61
	95% Lower Bound	98.868	142.50	168.46

Source: Authors' calculations and Québec Department of Higher Education.

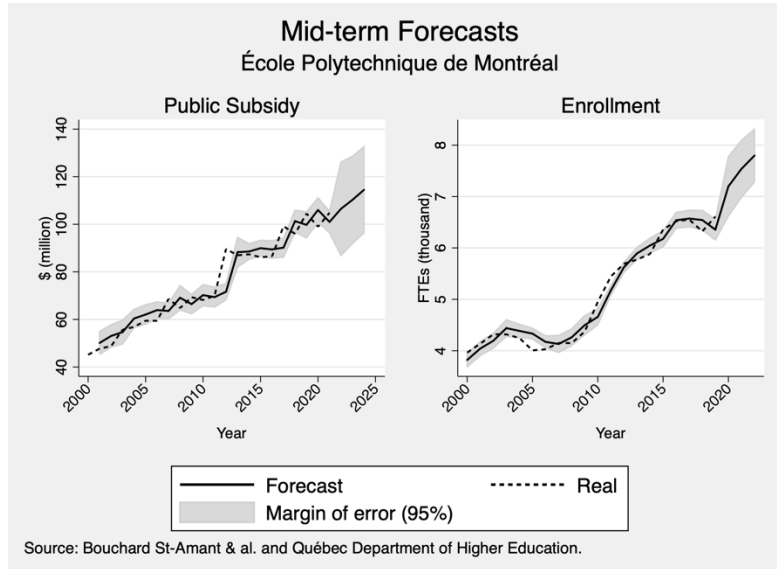


Figure A13. Polytechnique Montréal Forecast (Mid-term)

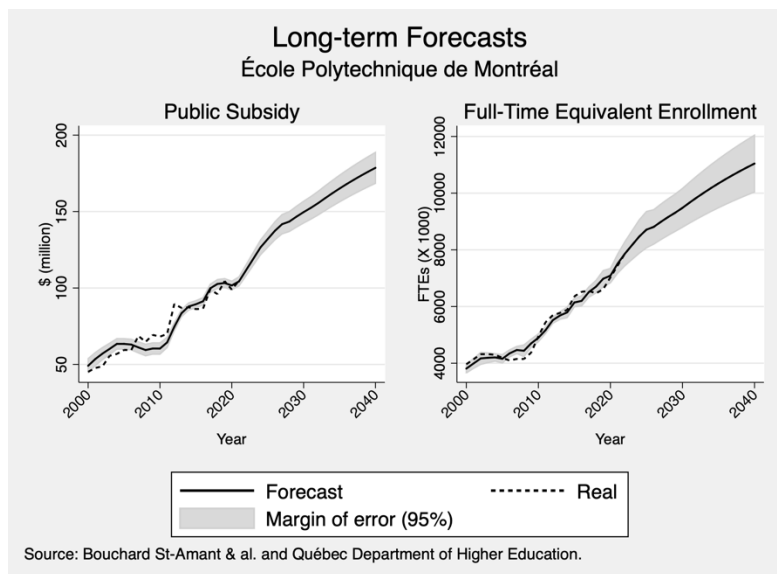


Figure A14. Polytechnique Montréal Forecast (Long-term)

$$F_t = 2.2841 + 0.7149 F_{t-1} + 0.0044 FTE_{t-1} + \varepsilon_t, \tag{A14}$$

$$FTE_{t-2} = -7604.566 + 0.7975 FTE_{t-3} + 0.001 pop_{t-7} + 134.43 u_{t-3} + \varepsilon_t. \tag{A15}$$

A.9 Université de Montréal

Table A15. Université de Montréal Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	38.300	43.340	45.084	46.193
	Forecast	38.115	42.055	43.846	44.909
	95% Lower Bound	36.995	40.770	42.609	43.625
Subsidy	95% Upper Bound	578.05	586.64	582.13	637.67
	Forecast	558.68	570.07	582.63	575.82
	95% Lower Bound	539.30	553.49	545.13	513.96

Source: Authors' calculations and Québec Department of Higher Education.

Table A16. Université de Montréal Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	41.880	49.674	54.039
	Forecast	40.709	47.524	51.036
	95% Lower Bound	39.537	45.376	48.034
Subsidy	95% Upper Bound	586.42	748.51	832.42
	Forecast	560.77	706.10	780.97
	95% Lower Bound	535.12	663.70	729.52

Source: Authors' calculations and Québec Department of Higher Education.

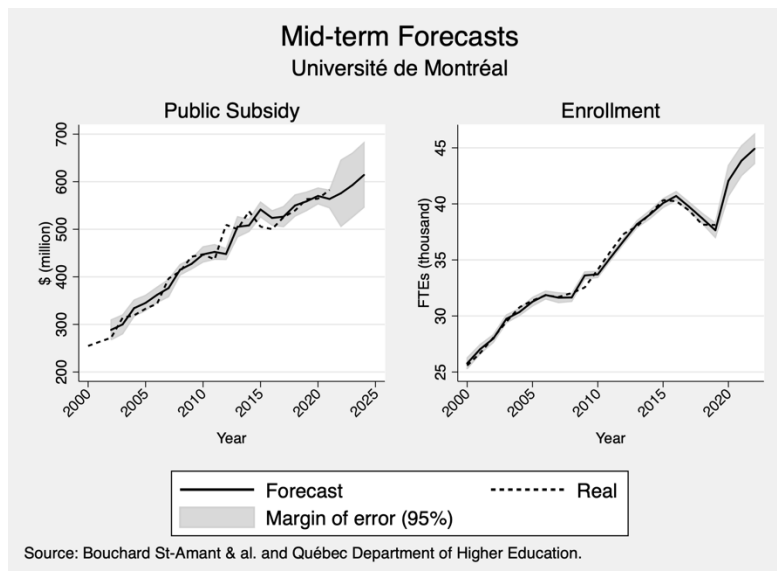


Figure A15. Université de Montréal Forecast (Mid-term)

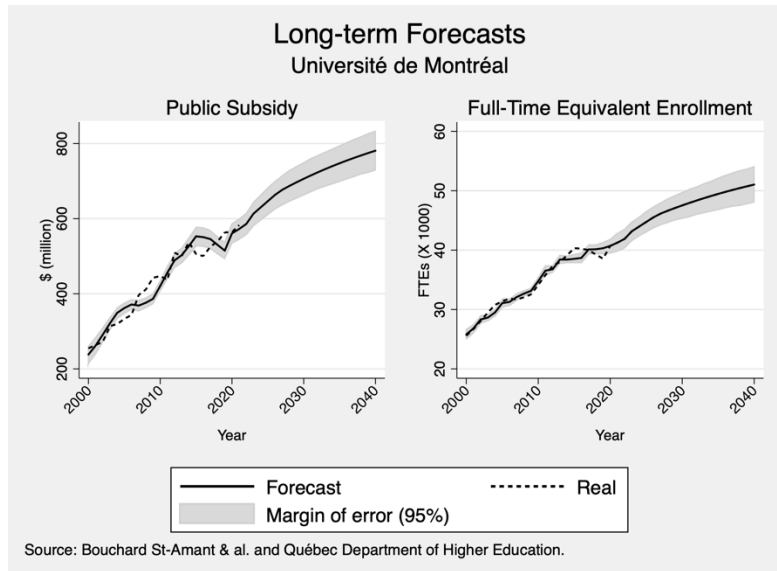


Figure A16. Université de Montréal Forecast (Long-term)

$$F_t = -21479.04 + 0.6658 F_{t-1} + \varepsilon_t, \tag{A16}$$

$$FTE_{t-2} = -35321.80 + 0.7855 FTE_{t-1} + 0.0046 Pop_{t-6} + 1006.78 u_t + \varepsilon_t. \tag{A17}$$

A.10 Université de Sherbrooke

See the main text for the short-term forecasting model and projections.

Table A17. Université de Sherbrooke Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	21.729	25.061	27.209
	Forecast	21.290	24.046	25.737
	95% Lower Bound	20.851	23.032	24.265
Subsidy	95% Upper Bound	300.63	377.17	418.49
	Forecast	290.03	360.75	398.81
	95% Lower Bound	279.43	344.34	379.14

Source: Authors' calculations and Québec Department of Higher Education.

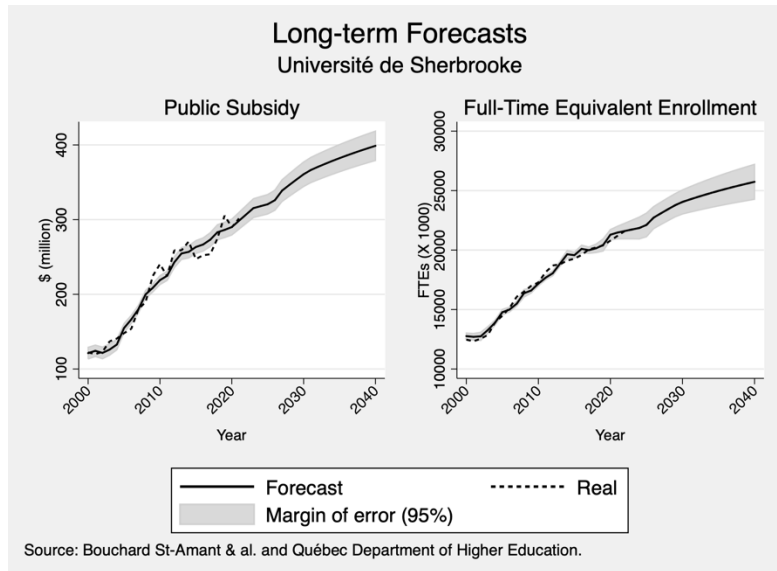


Figure A17. Université de Sherbrooke Forecast (Long-term)

$$F_t = -97.0369 + 0.0325 FTE_{t-1} - 0.0132 FTE_{t-2} + \varepsilon_t, \tag{A18}$$

$$FTE_{t-2} = -3188.33 + 0.9723 FTE_{t-3} + 0.0004 FTE_{t-4} + 125.8219 u_{t-3} + \varepsilon_t. \tag{A19}$$

A.11 Université Laval

Table A18. Université Laval Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	33.044	34.029	36.593	38.983
	Forecast	32.951	32.273	35.855	38.218
	95% Lower Bound	32.367	35.117	37.452	37.452
Subsidy	95% Upper Bound	468.35	489.05	472.48	517.34
	Forecast	447.54	471.42	465.11	462.84
	95% Lower Bound	426.73	453.77	441.06	408.33

Source: Authors' calculations and Québec Department of Higher Education.

Table A19. Université Laval Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	35.080	40.330	44.653
	Forecast	34.336	39.278	43.579
	95% Lower Bound	33.593	38.226	42.505
Subsidy	95% Upper Bound	465.74	591.68	702.38
	Forecast	444.40	551.57	645.05
	95% Lower Bound	423.06	511.68	587.61

Source: Authors' calculations and Québec Department of Higher Education.

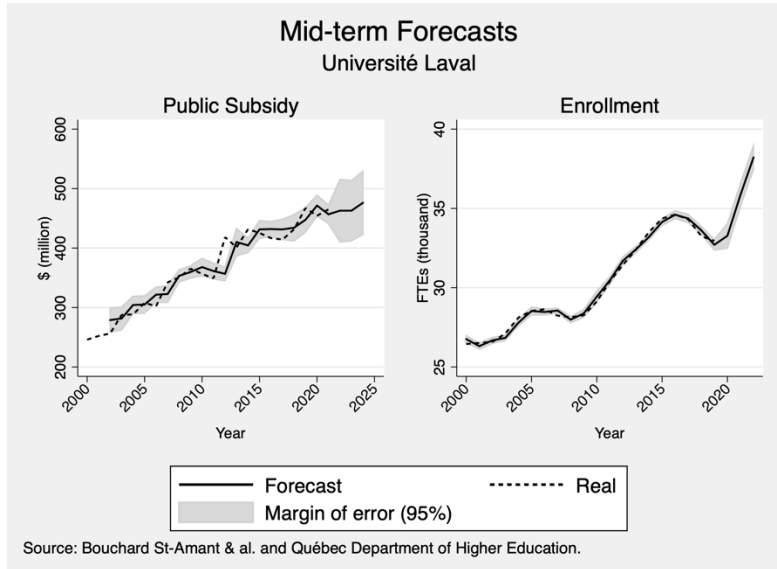


Figure A18. Université Laval Forecast (Mid-term)

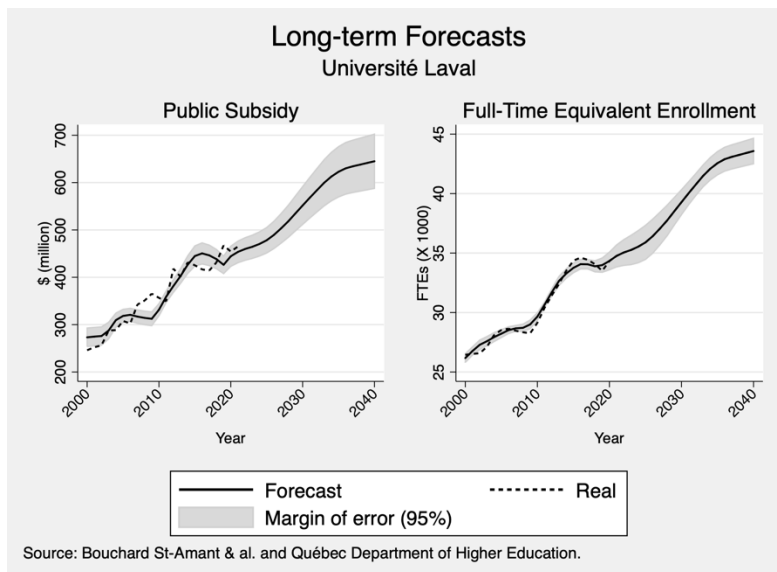


Figure A19. Université Laval Forecast (Long-term)

$$F_t = -35.576 + 0.0051 FTE_{t-3} + 0.7078 * F_{t-1} + \varepsilon_t \tag{A20}$$

$$FTE_{t-2} = -28841.98 + 388.41 u_{t-3} + 0.00543 pop_{t-5} + 1.0926 FTE_{t-3} - 0.6304 FTE_{t-4} + \varepsilon_t \tag{A21}$$

A.12 Université du Québec System

Table A20. Université du Québec System Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	70.309	70.884	70.669	69.538
	Forecast	67.314	67.965	67.792	66.775
	95% Lower Bound	64.319	65.046	64.915	64.011
Subsidy	95% Upper Bound	993.04	991.84	1010.23	1040.84
	Forecast	963.81	992.81	920.59	951.38
	95% Lower Bound	934.59	933.59	830.95	861.93

Source: Authors' calculations and Québec Department of Higher Education.

Table A21. Université du Québec System Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	71.600	84.959	95.787
	Forecast	69.693	82.413	93.206
	95% Lower Bound	67.786	79.866	90.624
Subsidy	95% Upper Bound	977.02	1270.23	1519.66
	Forecast	926.73	1186.25	1406.45
	95% Lower Bound	876.44	1102.29	1293.25

Source: Authors' calculations and Québec Department of Higher Education.

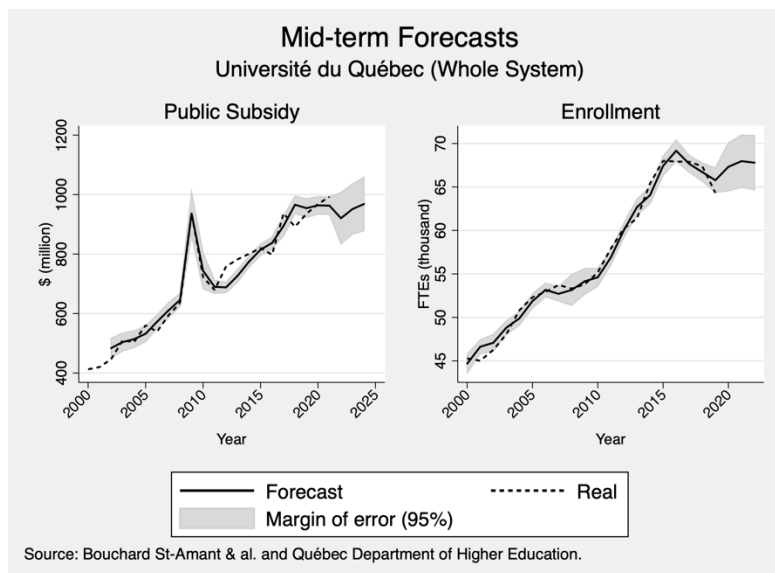


Figure A20. Université du Québec System Forecast (Mid-term)

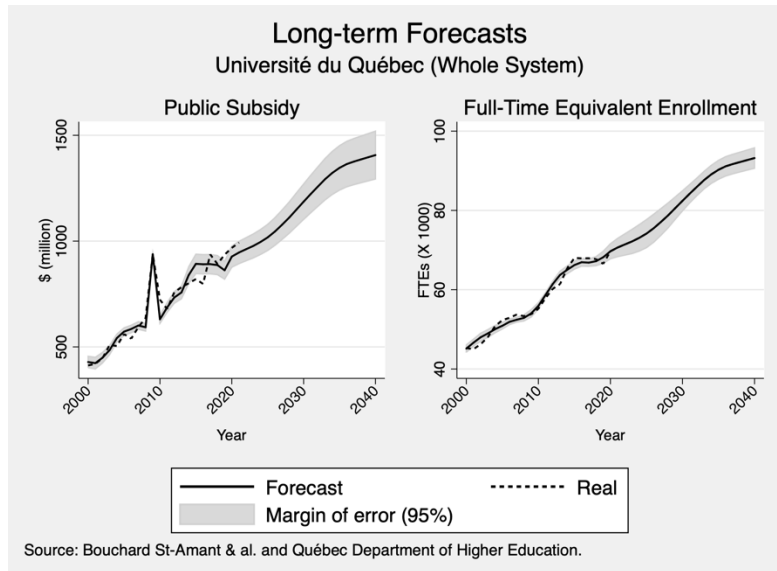


Figure A21. Université du Québec System Forecast (Long-term)

$$F_t = -299\,023 + 0.2204 F_{t-1} + 15.562 FTE_{t-3} + 271\,096 F_{2009=0} + \varepsilon_t \tag{A22}$$

$$FTE_{t-2} = -75\,114 + 0.0007 F_{t-3} + 0.78536 FTE_{t-3} + 0.0049 Pop_{t-6} + 0.5219 Pop_{18_{t-6}} + \dots + 963.76 u_{t-2} + \varepsilon_t \tag{A23}$$

A.13 Université du Québec à Chicoutimi

Table A22. Université du Québec à Chicoutimi Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	5.0763	5.0965	5.6110	5.6843
	Forecast	4.9828	5.0111	5.5200	5.5987
	95% Lower Bound	4.8894	4.9257	5.4289	5.5131
Subsidy	95% Upper Bound	92.534	88.291	95.611	95.215
	Forecast	87.878	85.317	84.864	84.825
	95% Lower Bound	83.224	80.661	74.117	74.436

Source: Authors' calculations and Québec Department of Higher Education.

Table A23. Université du Québec à Chicoutimi Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	5.3873	6.2055	6.7944
	Forecast	5.2813	6.0045	6.5101
	95% Lower Bound	5.1752	5.8034	6.2258
Subsidy	95% Upper Bound	89.135	117.91	139.87
	Forecast	84.425	109.54	128.60
	95% Lower Bound	79.716	101.17	117.33

Source: Authors' calculations and Québec Department of Higher Education.

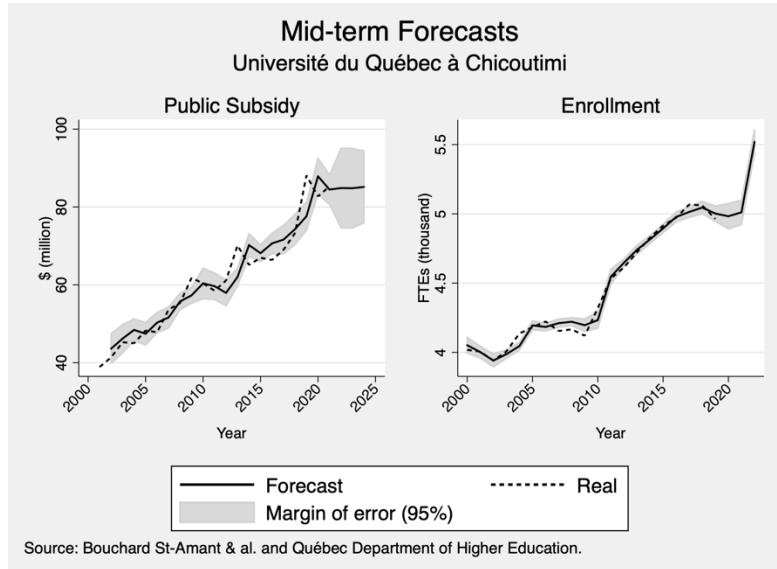


Figure A22. Université du Québec à Chicoutimi Forecast (Mid-term)

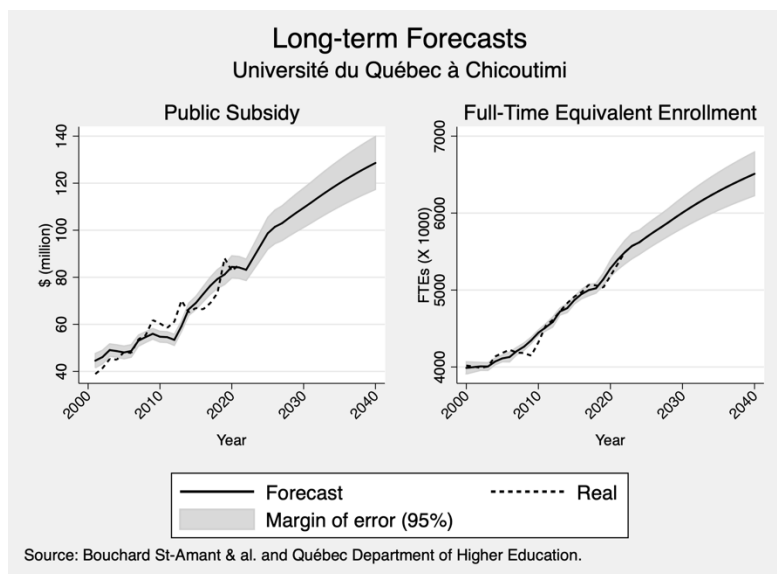


Figure A23. Université du Québec à Chicoutimi Forecast (Long-term)

$$F_t = -29.231 + 0.6105 F_{t-1} + 0.1251 FTE_{t-1} + \varepsilon_t \tag{A24}$$

$$FTE_{t-2} = -5681.291 + 0.3964 FTE_{t-1} + 0.001 Pop_{t-4} + 110.6882 u_{t-2} + \varepsilon_t \tag{A25}$$

A.14 Université du Québec à Montréal

Table A24. Université du Québec à Montréal Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	27.618	27.042	26.173	25.168
	Forecast	25.840	25.197	24.313	23.193
	95% Lower Bound	24.061	23.352	22.453	21.218
Subsidy	95% Upper Bound	325.86	299.14	326.12	322.70
	Forecast	313.35	321.04	272.67	268.93
	95% Lower Bound	300.85	278.55	219.23	215.15

Source: Authors' calculations and Québec Department of Higher Education.

Table A25. Université du Québec à Montréal Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	28.667	31.891	35.391
	Forecast	27.704	30.604	34.086
	95% Lower Bound	26.740	29.317	32.781
Subsidy	95% Upper Bound	316.72	415.48	535.43
	Forecast	293.93	373.05	468.04
	95% Lower Bound	271.14	330.62	400.66

Source: Authors' calculations and Québec Department of Higher Education.

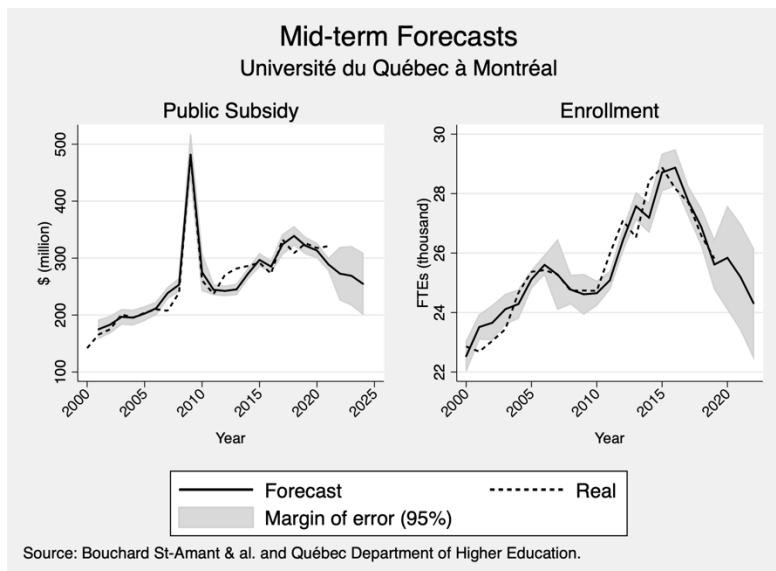


Figure A24. Université du Québec à Montréal Forecast (Mid-term)

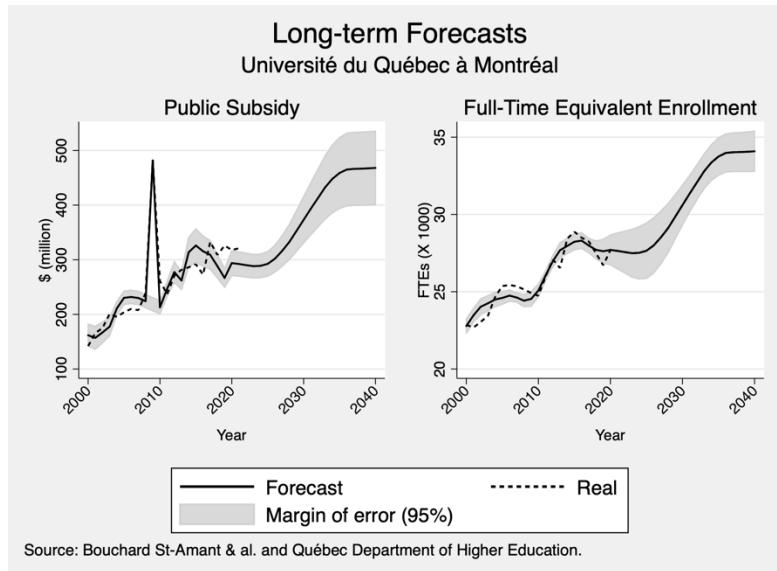


Figure A25. Université du Québec à Montréal Forecast (Long-term)

$$F_t = -305.5902 + 0.0873 F_{t-1} + 0.0213 FTE_{t-1} + 224.5116 F_{2009=0} + \varepsilon_t \quad (A26)$$

$$FTE_{t-2} = -19553.13 + 0.7859 FTE_{t-1} + 0.02874 Pop18_{t-4} + 110.6882 u_t + 258.6426 F_{2009} + \varepsilon_t \quad (A27)$$

A.15 Université du Québec à Rimouski

Table A26. Université du Québec à Rimouski Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	4.1275	4.5695	4.9806	5.2666
	Forecast	3.9199	4.2666	4.6534	4.9439
	95% Lower Bound	3.8211	3.9638	4.3261	4.6212
Subsidy	95% Upper Bound	72.743	69.349	65.985	71.005
	Forecast	68.705	65.825	73.242	60.045
	95% Lower Bound	64.666	62.300	59.847	49.086

Source: Authors' calculations and Québec Department of Higher Education.

Table A27. Université du Québec à Rimouski Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	4.8198	6.1169	7.0888
	Forecast	4.5607	5.8358	6.7342
	95% Lower Bound	4.3016	5.5547	6.3795
Subsidy	95% Upper Bound	69.893	96.341	123.95
	Forecast	64.791	86.184	108.41
	95% Lower Bound	59.689	76.026	92.876

Source: Authors' calculations and Québec Department of Higher Education.

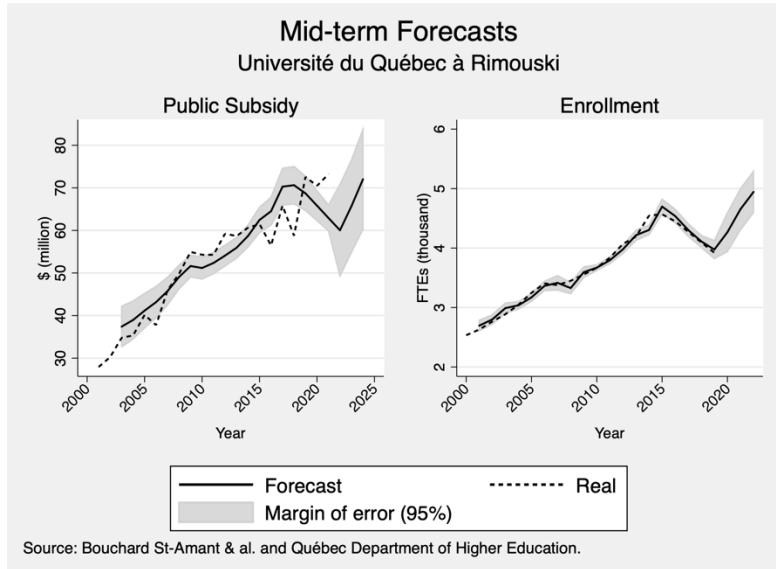


Figure A26. Université du Québec à Rimouski Forecast (Mid-term)

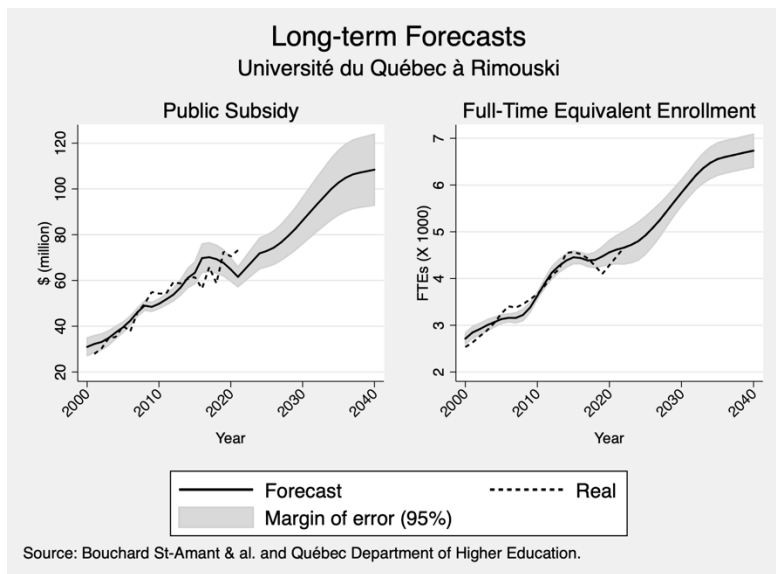


Figure A27. Université du Québec à Rimouski Forecast (Long-term)

$$F_t = -4010.347 + 16.3409 FTE_{t-3} + \varepsilon_t \tag{A28}$$

$$FTE_{t-2} = -4053.388 + 1.2214 FTE_{t-3} - 0.4488 FTE_{t-4} + 0.0005 pop_{t-6} + 94.821 u_{t-2} + \varepsilon_t \tag{A29}$$

Coefficients associated with u_{t-2} and FTE_{t-3} are not statistically different from zero.

A.16 Université du Québec en Abitibi-Témiscamingue

See the main text for the short-term forecasting model and projections.

Table A28. Université du Québec en Abitibi-Témiscamingue Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	2.4565	2.9074	3.1995
	Forecast	2.3631	2.8060	3.0716
	95% Lower Bound	2.2696	2.7046	2.9437
Subsidy	95% Upper Bound	47.015	69.587	75.243
	Forecast	45.103	65.760	70.916
	95% Lower Bound	43.191	61.933	66.589

Source: Authors' calculations and Québec Department of Higher Education.

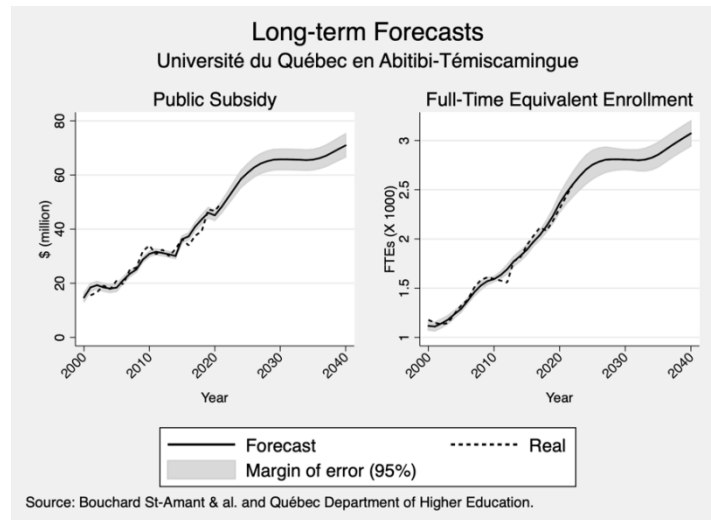


Figure A28. Université du Québec en Abitibi-Témiscamingue Forecast (Long-term)

$$F_t = -10.4856 + 0.0273 FTE_{t-1} + \varepsilon_t \tag{A30}$$

$$FTE_{t-2} = -2494.522 + 1.1001 FTE_{t-1} + 0.0004 Pop_{t-4} + 17.9446 u_t + \varepsilon_t \tag{A31}$$

A.17 Université du Québec en Outaouais

Table A29. Université du Québec en Outaouais Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	5.8302	5.6976	5.4484	5.1086
	Forecast	4.8599	4.7004	4.4536	4.1279
	95% Lower Bound	3.8896	3.7033	3.4587	3.1472
Subsidy	95% Upper Bound	80.350	74.261	88.624	88.292
	Forecast	76.021	73.394	69.371	69.136
	95% Lower Bound	71.692	66.959	50.118	49.981

Source: Authors' calculations and Québec Department of Higher Education.

Table A30. Université du Québec en Outaouais Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	5.5399	6.9233	8.0077
	Forecast	5.2449	6.5293	7.6083
	95% Lower Bound	4.9499	6.1353	7.2088
Subsidy	95% Upper Bound	75.787	96.500	123.72
	Forecast	72.529	91.475	116.18
	95% Lower Bound	69.270	86.450	108.64

Source: Authors' calculations and Québec Department of Higher Education.

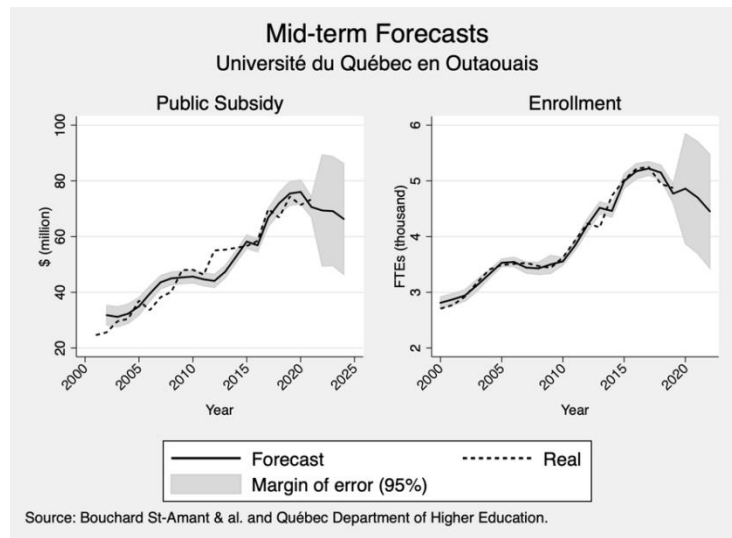


Figure A29. Université du Québec en Outaouais Forecast (Mid-term)

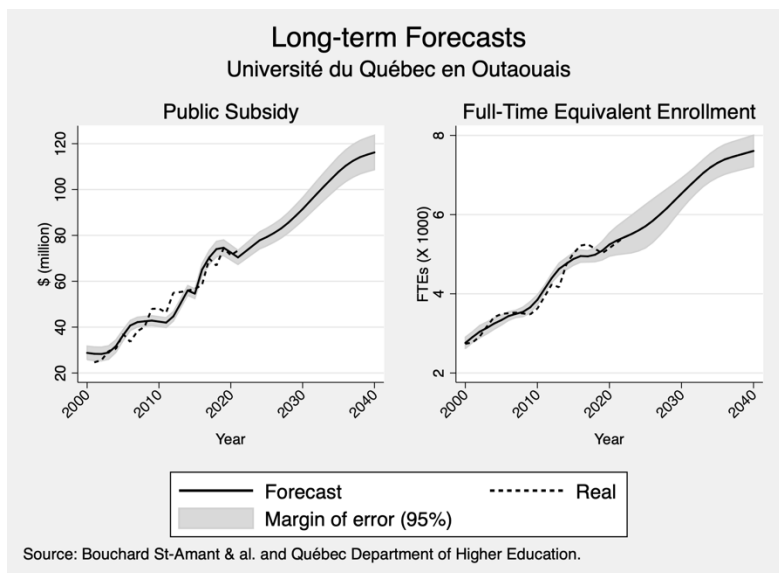


Figure A30. Université du Québec en Outaouais Forecast (Long-term)

$$F_t = -16.5344 + 0.0176 FTE_{t-1} + \varepsilon_t \tag{A32}$$

$$FTE_{t-2} = -6264.817 + 0.9847 FTE_{t-1} + 0.0074 Pop18_{t-3} + 55.3118 u_t + \varepsilon_t \tag{A33}$$

A.18 Université du Québec à Trois-Rivières

Table A31. Université du Québec à Trois-Rivières Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2020-2021	2021-2022
FTEs	95% Upper Bound	10.137	10.004	11.504	12.106
	Forecast	9.7008	9.5218	10.986	11.593
	95% Lower Bound	9.6801	9.0389	10.468	11.081
Subsidy	95% Upper Bound	136.67	144.62	139.98	144.55
	Forecast	131.11	138.35	137.84	137.84
	95% Lower Bound	125.54	132.08	129.62	117.68

Source: Authors' calculations and Québec Department of Higher Education.

Table A32. Université du Québec à Trois-Rivières Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	11.052	13.087	14.435
	Forecast	10.689	12.513	13.716
	95% Lower Bound	10.327	11.327	12.997
Subsidy	95% Upper Bound	138.85	182.60	210.60
	Forecast	132.50	171.83	196.88
	95% Lower Bound	126.15	161.06	183.17

Source: Authors' calculations and Québec Department of Higher Education.

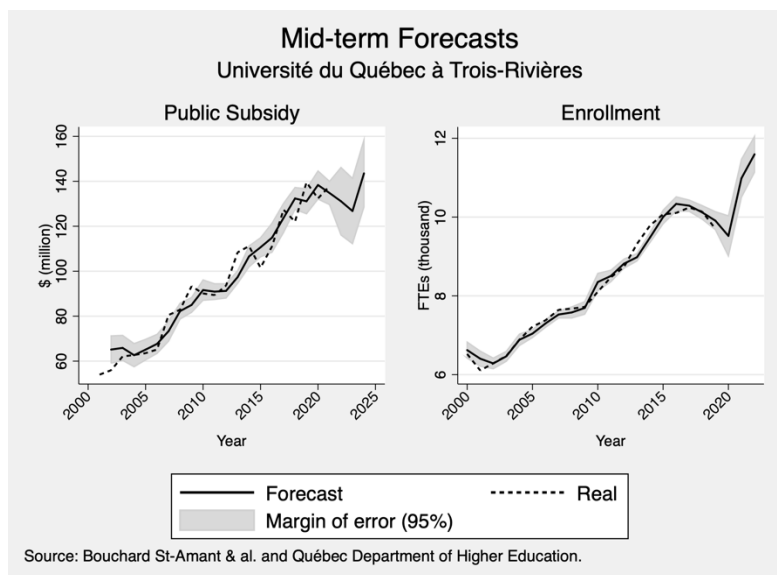


Figure A31. Université du Québec à Trois-Rivières Forecast (Mid-term)

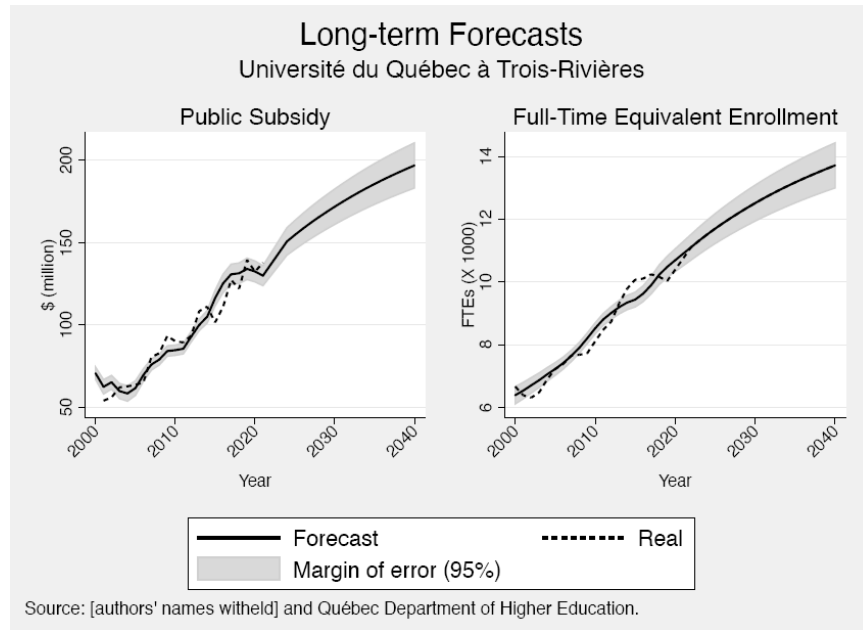


Figure A32. Université du Québec à Trois-Rivières Forecast (Long-term)

$$F_t = -32.591 - 0.0124 FTE_{t-3} + 0.0161 FTE_{t-4} + \varepsilon_t \tag{A34}$$

$$FTE_{t-2} = -14733.58 + 376.21 u_{t-3} + 0.0017 pop_{t-2} + 0.7634 FTE_{t-3} + \varepsilon_t \tag{A35}$$

A.19 T d é universit é

Table A33. T d é universit é Forecasting Results (Mid-Term)

		2018-2019	2019-2020	2021-2022	2021-2022
FTEs	95% Upper Bound	4.4423	4.8014	5.0131	5.0042
	Forecast	3.6691	4.2269	4.4678	4.4846
	95% Lower Bound	3.5616	3.6523	3.9225	3.9650
Subsidy	95% Upper Bound	37.357	40.959	54.539	42.803
	Forecast	35.123	38.173	43.589	34.010
	95% Lower Bound	32.889	35.387	43.869	25.216

Source: Authors' calculations and Qu ébec Department of Higher Education.

Table A34. T d é universit é Forecasting Results (Long-Term)

		2018-2019	2028-2029	2038-2039
FTEs	95% Upper Bound	5.1342	5.6919	6.3128
	Forecast	4.8229	5.3512	5.8630
	95% Lower Bound	4.5117	5.0104	5.4130
Subsidy	95% Upper Bound	54.180	63.973	73.450
	Forecast	44.473	51.304	57.923
	95% Lower Bound	34.767	38.685	42.350

Source: Authors' calculations and Qu ébec Department of Higher Education.

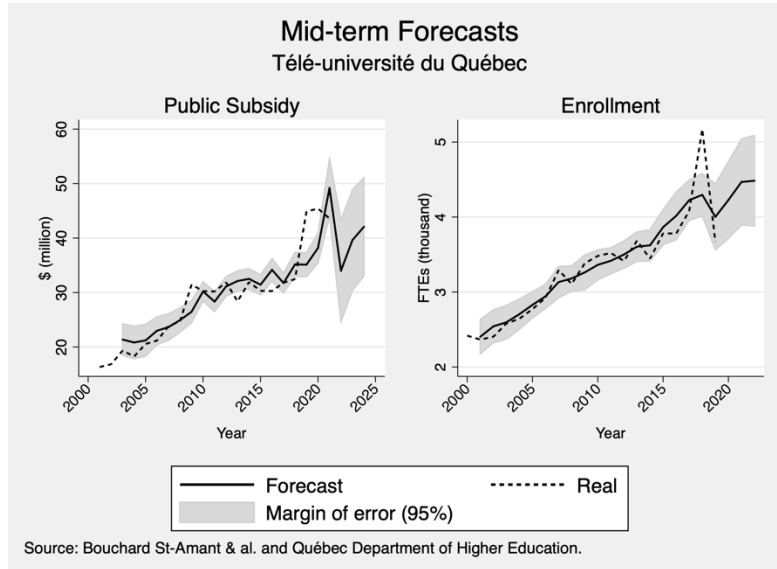


Figure A33. Télé-université Forecast (Mid-term)

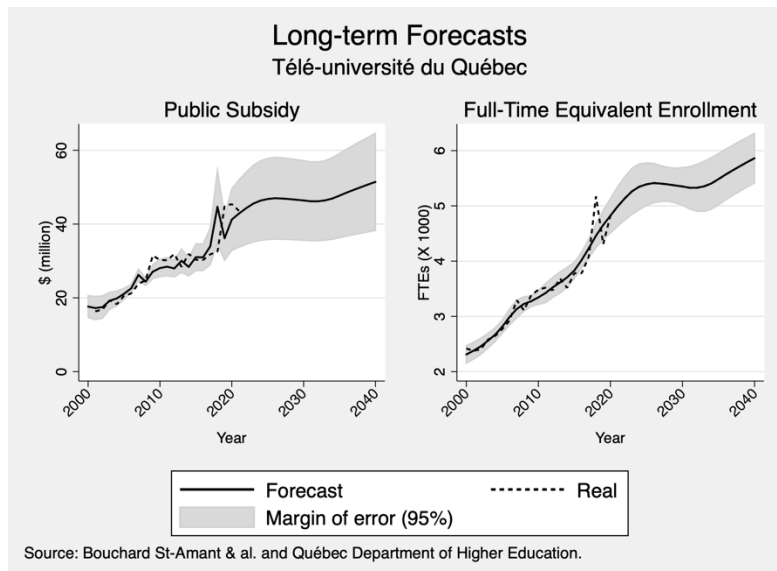


Figure A34. Télé-université Forecast (Long-term)

$$F_t = 23.110 - 0.0093 FTE_{t-3} + 0.0161 FTE_{t-4} + \varepsilon_t \tag{A36}$$

$$FTE_{t-2} = -13219.02 - 90.2535 u_{t-2} + 0.0023 pop_{t-2} - 0.3901 FTE_{t-3} - 0.1220 FTE_{t-4} + \varepsilon_t \tag{A37}$$

Appendix B

Linear Approximation of the Formula

Equation (1) can be approximated by (2) if each component is linearized. We focus on:

$$p_1 \max \left(\sum_j w_j FTE_{ijt-2}, \frac{1}{3} \left(\sum_j w_j FTE_{ijt-2} + \sum_j w_j FTE_{ijt-3} + \sum_j w_j FTE_{ijt-4} \right) \right) \tag{B1}$$

The other parts of (1) are linearized in a similar fashion. Suppose first that:

$$\sum_j w_j FTE_{ijt-2} \geq \frac{1}{3} \left(\sum_j w_j FTE_{ijt-2} + \sum_j w_j FTE_{ijt-3} + \sum_j w_j FTE_{ijt-4} \right) \tag{B2}$$

This then means that the component in (B1) amounts to:

$$p_1 \sum_j w_j FTE_{ijt-2} = p_1 \bar{w} FTE_{it-2} + p_1 \sum_j (w_j - \bar{w}) FTE_{ijt-2}, \tag{B3}$$

with $\bar{w} = \sum_j w_j FTE_{ijt-2} / \sum_j FTE_{ijt-2}$ being the average weight on FTEs. Then, with $\beta_1 = p_1 \bar{w}$ and $p_1 \sum_j (w_j - \bar{w}) FTE_{ijt-2}$, considered as an approximation error, the expression can be written as:

$$p_1 \sum_j w_j FTE_{ijt-2} = \beta_1 FTE_{it-2} + \epsilon_t, \tag{B4}$$

Now consider second that the inequality in (B2) is reversed. In such case, similar definitions for

$\bar{w}_2 = \sum_j w_j FTE_{ijt-3} / \sum_j FTE_{ijt-3}$, $\bar{w}_3 = \sum_j w_j FTE_{ijt-4} / \sum_j FTE_{ijt-4}$ and $\beta_i = \frac{1}{3} \bar{w}_i p_1$, and similar definitions for

the approximation errors, we have:

$$\begin{aligned} & \frac{1}{3} \left(\sum_j w_j FTE_{ijt-2} + \sum_j w_j FTE_{ijt-3} + \sum_j w_j FTE_{ijt-4} \right) \dots \\ & \dots = \beta_1 FTE_{it-2} + \beta_2 FTE_{ijt-3} + \beta_3 FTE_{ijt-4} + \epsilon'_t + \epsilon''_t + \epsilon'''_t \end{aligned} \tag{B5}$$

Combining the approximation errors into a single term yields the appropriate structure. Whichever direction is the inequality in (B3), both (B4) and (B5) are satisfied with (2). The other terms in (1) are linearized in a similar fashion.

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