Mathematical Understanding vs. Engineering Understanding: Engineering Students' Perceptions

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Abstract

The current study explores how BSc engineering students at an academic college of engineering perceive engineering and mathematical understanding and the interrelationships between them. The theoretical framework for this research includes three main aspects of engineering and mathematical understanding: procedural, conceptual, and applicable. The participants were thirty BSc students from different engineering disciplines who answered a four-open-items questionnaire that included three questions dealing with specific mathematical and engineering subjects and one general question. Content analysis of the students' answers revealed that all three aspects were reflected in the students' answers. More responses were recognized in student answers to the specific questions than to the general question. The procedural aspect was very prominent among the students' responses to the specific mathematics and engineering subject. Regarding the answers to the general question, it can be induced that students possess general perceptions of mathematic understanding as procedural and conceptual, but not applicable; and engineering understanding as conceptual and applicable, but not procedural. Concerning relationships between mathematical and engineering understanding, more than one third of the students claimed that mathematics is a tool for engineering; yet, at the same time, not even one student addressed applicable aspects of mathematical understanding in the general question. This fact stresses the students' detached general perception of mathematical understanding as not applicable.

Keywords: mathematical understanding, engineering understanding, student perceptions

1. Introduction

The 21st century is characterized by information explosion; nevertheless, accessibility to information is friendly and becoming easier as time advances. Therefore, teaching for understanding has become the trend, rather than teaching information. The subject of mathematics has always existed at the core of engineering education. Engineering students take many courses in mathematics while qualifying for a first academic degree in engineering. Nevertheless, a thorough literature review of academic journals and using Google Scholar revealed no articles regarding engineering students' perceptions about the nature of mathematical understanding and engineering understanding, considering the similarities and the differences between them. The authors of the current paper find it interesting to investigate the students' perception of mathematical understanding versus engineering understanding and the relationship between them.

The paper is organized as follows. First comes the literature review about understanding in general, then mathematical understanding is followed by the literature review about engineering understanding. Secondly is the research plan, including the research questions, population, and methodology, and finally come the findings followed by a discussion and conclusion.

2. Literature Review

2.1 Approaches to Understanding

According to the Merriam-Webster dictionary (2016), understanding is "the capacity to apprehend general relations of particulars". Harpaz (2013) argues that to understand something means to relate it to something else. For example, if we are stuck in a traffic jam we might relate the cause to the traffic lights or to an accident. The same author also suggests (2015) that knowledge is exploding, accessible, and becoming obsolete. Therefore, it is better to teach students to deal with knowledge, i.e. to think, rather than to teach knowledge. He also states (ibid, p. 38) that "there is an intimate connection between thinking and knowledge, and when knowledge is understood, thinking is conducted flexibly."

To explain the term of understanding, Perkins and Blythe (1994) suggest the “performance perspective,” as being able to do a variety of thought-demanding things with a topic—like explaining, finding evidence and examples, generalizing, applying, analogizing, and representing the topic in a new way. Harpaz (2009) expands the approach suggested by Perkins and Blythe into 18 understanding performances, among them: explaining knowledge, commenting knowledge, offering a model for knowledge, representing knowledge in different ways, taking knowledge apart, joining knowledge to a big picture, applying knowledge to different contexts, making comparisons, making distinctions, making generalizations, asking questions about knowledge, and creating new knowledge. Perkins and Blythe (1994) state that there is no such thing as general good thinking; there is only good thinking in a particular domain; a domain that the thinker understands (his or her cognitive comfort zone). Rittle-Johnson (2006, p. 2) offers a definition of conceptual understanding that complements the definition of Perkins and Blythe: “understanding of principles governing a domain and the interrelations between units of knowledge in a domain.”

2.2 Mathematical Understanding

Many researchers in the area of mathematical education emphasize the importance of understanding. Skemp (1976) argues that to understand a concept is to place it in the appropriate scheme of the mental structure in which one's knowledge is organized. Hiebert & Carpenter (1992) claim that understanding occurs when different representations of a subject become connected into a large and well-organized network. To understand a mathematical item in a deep way, according Michener (1978), “one must know about the item itself; its intra-space relation to other items of the same type; its inter-space relation to other items of different types; its dual relation to other items of like type; and its relations to items in other theories” (p. 378). Michener also emphasizes the importance of understanding the connections between different mathematical concepts and ideas, as well as the ability to perceive the greater context of the concept or theory.

Skemp (1976) distinguishes between “instrumental understanding”, which is knowing what to do, and “relational understanding”, which is knowing why. Similar concepts can be found in Hiebert (1986), who relates to procedural versus conceptual knowledge. The lines separating these two types of knowledge are not conclusive. He defines conceptual knowledge as a connection-abundant type of knowledge. Accordingly, a piece of information is only considered as a part of the learner's conceptual knowledge if he or she recognizes its relation to the other information fragments connected to it. Procedural knowledge contains symbolic representations of concepts and subjects, rules, algorithms, and procedures, which come into use in problem solving. Some researchers (Kilpatrick, Swafford, & Findell, 2001; Schneider, Rittle-Johnson, & Star, 2011) argue that these two kinds of knowledge are important in learning and understanding mathematical subjects and hypothesize that both types of knowledge contribute to the ability to solve problems in flexible and efficient ways. Schneider, Rittle-Johnson, & Star (2011) found that conceptual and procedural knowledge contribute to procedural flexibility, which is the ability to solve a problem in more than one way. This meets Perkin's and Blythe suggestion (1994) to perceive understanding as being able to do a variety of thought-demanding things with a topic.

Ma (1999) referred to profound understanding of mathematics as to coherent knowledge, and understanding the interconnections between different subjects, topics, and principles; using different strategies and multiple approaches to solve a given problem. She defined two kinds of understanding: understanding a topic with depth as connecting it with more conceptually powerful ideas on the subject, and understanding a topic with breadth as connecting it with similar or less conceptual power. Depth and breadth depend on thoroughness—the capability to "pass through" all parts of the field—to weave them together.

2.3 Engineering Understanding

In contrast to mathematics pedagogical research, engineering pedagogical literature pays less attention to the question: what is engineering understanding? The most comprehensive reference to the issue was found in the internet site “What is Engineering” (2016), written by experienced engineers from industry and academia, who described their point of view of engineering understanding as composed of three basic principles: 1) conceptual understanding, which is the ability to explain theory and thereby to reinforce personal knowledge and deep individual understanding; 2) understanding the mathematics behind an engineering concept; 3) recognizing the practical applications of engineering concept and the ability to apply it in real life.

A number of authors point out the importance of mathematical understanding for engineers. Thus, Cox (2001) emphasizes that even though most engineers use mathematics as a tool, they need to have an idea of how and why the tool works. According to him, mathematical knowledge donates to deeper understanding of engineering concepts.

The report of the National Academy of Engineering and National Research Council (2009) describes a broad picture of relationships between engineering and science. It points out that engineering is intimately related to science and mathematics. In every field of engineering, an understanding of the relevant science is a prerequisite to understanding engineering concepts and practical work. Thus, chemical engineers must understand chemistry, bioengineers must understand molecular biology, electronics engineers must understand the physics of semiconductors, and so on.

A number of authors highlight the importance of conceptual understanding. Thus, Streveler et al. (2008) emphasize that understanding conceptual knowledge is critical to the development of competence in engineering students and in practicing professionals. Savander-Ranne and Kolari (2003) claim that mastery of relevant concepts and phenomena generates a necessary base for the acquisition of knowledge and understanding in engineering subjects.

Additionally, Savander-Ranne and Kolari (2003) stressed the difference between procedural and conceptual understanding. They point out that the ability to solve numerical problems and handle algorithms is no proof of conceptual understanding. Technique in mathematics is necessary, but it does not display the conceptual difficulties of an issue and how a student is able to cope with these difficulties. Nurrenbern and Pickering (1987) report that students who solved numerical problems correctly were not necessarily able to solve conceptual problems. They found that students rely more on algorithmic techniques than on reasoning skills.

The most important engineering activity is design (Simon, 1997), which is the development of new products, devices, and systems. Dym et al. (2005) define engineering design as a "systematic, intelligent process in which designers generate, evaluate, and specify concepts for devices, systems, or processes" (p. 104). In the design process, engineers integrate various skills and types of thinking. According to Eide, Jenson, Mashaw, and Northrup (2002), engineers use mathematics knowledge, scientific principles, and practical heuristics. An engineer needs a detailed understanding of the designed part of the device, and a holistic or systems understanding of the completely designed system. Based on the literature above, it can be claimed that all these various abilities set up the applicable aspect of engineering understanding.

2.4 Engineering Students' Conceptions of Understanding in Mathematics

The report of The National Academy of Engineering and National Research Council states that every engineering student is equipped with a rich mathematical toolbox during the student's education. The report claims: "Mathematics is as fundamental to engineering as science. Engineers use mathematics both to describe data (e.g., graphs showing the strength or other properties of a material under varying conditions) and to analyze them (e.g., the flow rate of fluids through the pipes of a chemical plant). Engineers use science and mathematics most obviously in building and analyzing models" (Engineering in K-12 Education: Understanding the Status and Improving the Prospects, 2009, p. 43). It is important to comprehend how engineering students perceive the concepts of mathematical and engineering understanding. The literature review of academic journals in the area revealed very few articles regarding engineering students' perceptions of the nature of mathematical understanding and no references to students' perceptions of engineering understanding. This study seeks to fill the gap in the literature.

The most extensive reference to the issue was established in the research of Khiat (2010). The research addresses engineering students' conceptions of understanding in mathematics and suggests a theory of engineering mathematics understanding that comprises conceptual, functional, procedural, disciplinary, and associational understanding. In the research context, conceptual understanding refers to the ability to understand how mathematical formula are derived. Functional understanding refers to the comprehension of the functions of the mathematical formula and procedures in the mathematics domain. Procedural understanding refers to the ability to model the steps in the mathematical formula when solving mathematical problems. Disciplinary understanding refers to the ability to understand engineering concepts and theories in engineering and knowing the various engineering concepts and theories when solving engineering problems. Associational understanding refers to the ability to relate and utilize the mathematical formula for the engineering problems they are tackling.

The results of the study (ibid) showed that engineering students generally focused on achieving procedural and associational understanding of engineering mathematics. Conceptual understanding in engineering mathematics learning was ignored. One of the explanations of the findings was the students' feeling that conceptual understanding is irrelevant to them, as they are not required to prove or justify the formula they are expected to learn in their future engineering careers.

2.5 Theoretical Framework of the Study

The research focused on three main aspects of engineering and mathematical understanding. The procedural aspect, including mainly technical performance and accomplishing procedures, corresponds to Skemps' (1976) instrumental understanding, to Hieberts' procedural knowledge (1986), and to Khiats' (2010) procedural understanding. The conceptual aspect, especially recognizing various connections, corresponds to Skemps' relational understanding and to Hieberts' conceptual knowledge. The conceptual aspect in the research is related in both fields (mathematics and engineering), thus it includes Khiats' conceptual understanding, functional understanding, and disciplinary understanding. The applicable aspect of understanding was suggested for the research and mainly stresses the ability to apply the knowledge in different fields, such as everyday life or the engineering area. This aspect particularly corresponds to Hieberts' conceptual knowledge, Skemps' relational understanding, and Khiats' associational understanding. We believe that applying knowledge within a completely different area requires more than just recognizing connections, but the ability to solve new real life problems. In addition, this aspect is particularly important when it comes to engineering. A detailed description of the aspects appears in Table 1 in "The Data Analysis" section.

3. Method

3.1 Research Objective and Questions

The aim of the study is to explore how BSc engineering students at an academic college of engineering perceive engineering and mathematical understanding.

The research questions are:

1. How do engineering students perceive mathematical understanding?
2. How do engineering students perceive engineering understanding?
3. How do students perceive the interrelationships between mathematical understanding and engineering understanding?

3.2 Population

The research population consists of thirty students (S1–S30) at an academic college of engineering, who studied the course "Technique, mathematical understanding, and what is between them", as part of a General Studies program, during the first semesters of the academic years 2015 and 2016. The participants came from engineering departments of electrical and electronic engineering, software engineering, industrial engineering and management, biotechnology engineering, and mechanical engineering. The students' ages ranged from 20 to 25 years old.

3.3 Research Methodology and Tools

Qualitative methodology based on content analysis of the students' answers to a four-open-items questionnaire was used in the study. The main research tool is the questionnaire.

The students were asked to write about their perception of the terms "mathematical understanding" and "engineering understanding" before they start studying the course. The questions included in the questionnaire were the following:

Q1) In the calculus lesson today, we learned about the "limit of a function". I do not understand the term, and am very frustrated about that. My roommate, Dani, who will graduate from the school of engineering at the end of the year, claims that he understands it perfectly and can help me. What questions should I ask Dani and what aspects of the term should I examine to verify that Dani indeed understands the subject perfectly? Please address as many aspects as you can and elaborate.

Q2) Give an example of a mathematical subject that you understand. How do you know that you understand it, and what, in your opinion, helped you understand it?

Q3) Give an example of an engineering subject that you understand. How do you know that you understand it, and what, in your opinion, helped you understand it?

Q4) What, in your opinion, is mathematical understanding and what is engineering understanding? Is there a similarity/difference between the two? Please explain and elaborate.

The specific questions Q1 to Q3 deal with the students' perceptions of mathematical and engineering understanding while addressing a specific mathematical/engineering subject. The more general question, Q4, deals with students' declared perceptions of mathematical vs. engineering understanding and students' perceptions of interrelationships between mathematical and engineering understanding.

3.4 Data Analysis

Three experts categorized the students' answers according to three aspects of understanding: procedural, conceptual, and applicable. In every case of disagreement, the experts continued to discuss until they reached agreement. The experts include someone with 43 years’ experience (30 of them in academia) in electrical and electronic engineering and a PhD in engineering education; someone with 39 years’ experience (25 of them in academia) in electrical and electronic engineering and a PhD in engineering education; and someone with 18 years’ experience in academia and a PhD in mathematics education.

Table 1 depicts the aspects of understanding accompanied by typical quotes from the students’ questionnaires with the goal to justify the categorization. Additional quotes of students’ answers corresponding to understanding different aspects are presented in the “Findings” section. The corresponding student number (S1-S30) and question number (Q1-Q4) appear next to each quote.

Table 1. The aspects of mathematical and engineering understanding in data analysis

Aspect	Characterization	Typical quote	Justification of Characterization
Procedural	To know rules, laws, algorithms, and procedures; to know how to technically perform, solve, or accomplish a procedure, exercise, or task; to recognize symbols connected to the concept, and to know “how to do” something.	In my opinion, mathematical understanding is more technical. They give you an exercise that has a very technical way to solve it (substituting in the equation). I solve it like a robot, just like I learned in class (S7, Q4).	The word “robot” in the answer emphasizes the technical aspect of the mathematical understanding.
Conceptual	To understand “why”, to recognize connections, different representations, and relationships, and to understand/generate different strategies/ways to solve problems.	Mathematical understanding is understanding how to get the particular solution; where the solution came from and not just the calculations (S18, Q4).	The student's relation to both practical aspect of calculation and to the reason of the use of this particular calculation
Applicable	To be able to apply the knowledge outside the specific context, in everyday life or in another field or area.	I know that I understand the subject because I can see its use in real life (S10, Q3).	The reference to the use of the subject in real life is stressed in the answer.

The analysis revealed different students’ approaches to mathematical and engineering understanding as well as interrelationships between them. Some of the students’ responses include several aspects of understanding, therefore the total number of understanding aspects is greater than the number of student responses.

4. Findings

The analysis of students' answers to the questionnaire indicates different approaches of students to specific questions compared to their answers to the general question. Therefore, this section presents the two issues separately.

4.1 Students’ Perceptions of Mathematical and Engineering Understanding while Dealing with a Specific Subject

The analysis of students’ perceptions of mathematical and engineering understanding while dealing with a specific subject (Q1–Q3) is presented in the following section.

4.1.1 Perceptions of Mathematical Understanding

The data presented here were collected from students' responses to questions Q1 and Q2. One question dealt with the "limit of a function" (the Dani question, Q1) and the other question requested the students to give an example of a mathematical subject they understand (Q2).

Twenty-three of thirty answers for Q1 and twenty-four of thirty answers for Q2 included the *procedural aspect* of *mathematical understanding*. The following citation represents the procedural aspect of mathematical understanding:

"To make sure that Dani understands, I would ask him how we find a limit of a function, and what are the steps for finding the limit." (S2, Q1)

Nineteen of thirty answers for Q1 and twelve of thirty answers for Q2 included the *conceptual aspect* of *mathematical understanding*.

A typical answer dealing with the conceptual aspect of mathematical understanding:

"Different representations of the subject such as algebraic presentation and graphic presentation help [to understand]". (S18, Q2)

Thirteen of thirty answers to Q1 and seven of thirty answers to Q2 addressed the *applicable aspect* of *mathematical understanding*.

One student relates to the applicable aspect of mathematical understanding as follows:

"To make sure that Dani understands, I would ask him if there is a way to demonstrate the limit of function in practice….. to give a concrete example where a limit of function exists in everyday life." (S23, Q1)

4.1.2 Perceptions of Engineering Understanding

This section presents the students' perceptions of engineering understanding while dealing with a specific engineering subject (Q3). It should be noted that it was impossible to address one common subject for all the students (similar to the Dani question), because the students came from different engineering programs.

The *procedural aspect* of *engineering understanding* was found in fifteen of thirty answers.

An example of the procedural aspect in an answer dealing with engineering understanding:

"Solving a lot of exercises on the subject helps me [to understand]." (S7, Q3)

Ten of thirty answers related to the *conceptual aspect* of *engineering understanding*. A typical quote:

"The very fact that we deal with the subject from a theoretical point of view—connecting the mathematical to the engineering aspects, performing experiments and designing a system—has contributed to my understanding." (S3, Q3)

The mention of the *applicable aspect* in *engineering understanding* was found in nine of the thirty answers. An example:

"Seeing the subject in reality and in real life helps me understand it." (S23, Q3)

Table 2 summarizes the numbers of students' answers, referring to the specific questions (Q1-Q3) in mathematical and engineering understanding.

Table 2. Summary of students' perceptions of mathematical and engineering understanding while dealing with a specific subject

	Understanding		
Aspect	**Mathematical understanding**		**Engineering understanding**
	Q1	**Q2**	**Q3**
Procedural	23	24	15
Conceptual	19	1	10
Applicable	13	7	9
Total	55	32	34

4.2 Students' Declared Perceptions of Mathematical and Engineering Understanding

In this section, we present the students' perceptions of mathematical and engineering understanding that were demonstrated in their answers to the general question (Q4): "What, in your opinion, is mathematical understanding and what is engineering understanding? Are there similarities/differences between the two? Please explain and elaborate." It is important to note that a response of each student can relate to both the mathematical and engineering understandings and also to the interrelationships between them. Additionally, one response can relate to several aspects of understanding (procedural, conceptual, or applicable).

4.2.1 Perceptions of Mathematical Understanding

Seven of thirty answers included references to the *procedural aspect* related to *mathematical understanding.* A typical quote:

"Mathematical understanding is converting each problem to an equation and solving it." (S10, Q4)

Fourteen of thirty answers addressed the *conceptual aspect* related to *mathematical understanding*. An example of this aspect in the answers:

"Mathematical understanding is deep understanding of mathematical subjects, understanding of the meaning of the subject." (S13, Q4)

4.2.2 Perceptions of Engineering Understanding

In nine answers the *conceptual aspect* appeared, related to *engineering understanding.* An example of this aspect in the answers:

"An engineering problem is more complicated; it isn't unequivocal or has one and only one answer. In engineering there are always several ways to find a solution and we need to find the best or most effective or cheapest or simplest or fastest way." (S2, Q4)

Nine answers included the *applicable aspect* related to engineering understanding. A typical qoute:

"Engineering understanding is the ability to take the theory and use it to create a real artifact in the world." (S27, Q4)

Table 3 depicts the distribution of students' answers to Q4, relating to aspects of mathematical and engineering understanding.

Table 3. The summary of students' declared perceptions of mathematical and engineering understanding

	Understanding	
Aspect	**Mathematical understanding**	**Engineering understanding**
Procedural	7	
Conceptual	14	9
Applicable		9
Total	20	18

4.3 Students' Perceptions of Interrelationships between Mathematical and Engineering Understanding

Six of thirty students emphasized the *similarity* between mathematical and engineering understanding. Following is an example of such an answer:

"The mathematical understanding and the engineering understanding are totally the same; it's problem solving, and doesn't matter what the area is." (S16, Q4)

Seventeen students emphasized the differences between the two kinds of understandings. A typical quote:

"There is a difference between the two. The mathematical understanding is in-depth theoretical understanding and investigation. Engineering understanding is practical." (S20, Q4)

Nine answers included this statement: *"Mathematics is a basis/a tool for engineering."* Following is an example of such an answer:

"There is a tight connection between the two, because to understand the practice and application, there is a need for theoretical mathematical understanding, and the majority of engineering theory is mathematical-numerical in its basis." (S23, Q4)

Table 4 summarizes student perceptions about the interrelationships between mathematical and engineering understanding

Table 4. Students' perception of the interrelationship between mathematical and engineering understanding

	Similar	**Different**	**Mathematic is basis for engineering**
No. of students	6	17	9

5. Discussion and Conclusion

The main issue in the current research is the students' perceptions of mathematical and engineering understanding. Nevertheless, the research results show different approaches of students to specific questions in comparison with the general question. Therefore, the beginning of the discussion deals with an explanation of this phenomenon, which helps to interpret the study results.

The research results (Table 2) show that all three aspects were reflected in students' answers dealing with mathematic and engineering understanding. Regarding the *specific questions* (Q1-Q3), the results show that a majority of students' responses (55) appeared in Q1, which is the only question including a well-defined scenario about understanding of a mathematical subject, while in Q2 and Q3 the students were asked to offer their own examples of mathematical and engineering subjects; they offered only 32 responses concerning mathematical understanding (Q2) and 34 responses dealing with engineering understanding (Q3). Concerning answers to the *general question* (Q4), 20 responses about mathematical understanding and 18 responses about engineering understanding were collected (Table 3). It is noticeable that more responses were found when dealing with specific questions than with the general question. The general question was presented to the students after the specific questions. Despite that, it seems that recognizing different understanding aspects in specific subjects did not help them to answer the general question. The possible explanation for these results is that it is easier for students to relate to understanding a well-defined case than to give their own examples. Moreover, the toughest case for students is to relate to understanding when answering the general question, which does not include any specific subject, but deals with understanding in general terms. This explanation is in line with the research results of Chou and Chen (2016), who examined epistemological beliefs about certainty, simplicity, source, and justification of engineering knowledge of electrical engineering students. They concluded that the engineering students have slightly sophisticated beliefs about general concepts.

The first research question deals with engineering students' perceptions of mathematical understanding. Three questions of the questionnaire were designed to collect the students' perceptions: Q1 and Q2 with *specific questions* (Table 2), and Q4 with a *general question* (Table 3). Regarding the *understanding of a specific subject*, the students mentioned all three aspects of *mathematical understanding*. The *procedural aspect* is very prominent in the answers. This aspect was found in 23 and 24 responses for Q1 and Q2, respectively. The *conceptual aspect* was found in 19 responses and in one response, for Q1 and Q2, respectively. The *applicable aspect* appeared in 13 and 7 responses, for Q1 and Q2, respectively. When dealing with the *general question* (Q4), the students related only 7 times to the *procedural aspect* and 14 times to *conceptual aspects* (Table 3). It is interesting that the *applicable aspect* of mathematical understanding was not mentioned at all in the answers to the general question, even though it appeared in the answers dealing with the specific subject. The results above partially correspond to the results of Khiat (2010), who states that engineering students referred to *procedural understanding* of engineering mathematics and ignored *conceptual understanding*.

The second research question deals with engineering students' perception of *engineering understanding*. Question Q3 of the questionnaire, which is a *specific question,* and Q4, which is a *general question,* were designed to aggregate the students' perception of engineering understanding. Regarding the *specific question* (Q3), all three aspects were mentioned: *procedural aspect* – by 15 students, *conceptual aspect* – by 10 students, and *applicable aspect* – by 9 students (Table 2). Nevertheless, when it came to the general question (Q4), the students only mentioned the *conceptual aspect* and the *applicable aspect* (nine appearances each) (Table 3). It is interesting that even though the *procedural aspect* of engineering understanding was mentioned by the students 15 times in the specific question (Q3), it was not mentioned at all in students' answers to the general question (Q4).

It can be summarized that students’ general perceptions of mathematical understanding are technical (the procedural aspect) and theoretical (the conceptual aspect), but not applicable. Likewise, from a general point of view, the students recognize engineering understanding as theoretical and applicable, but do not see its procedural aspect. It can be assumed that they perceive mathematics as detached from real life, while engineering as a creative process lacking routine and everyday repetitive work. These naïve beliefs reflected in the answers to the general question differ from their more sophisticated perceptions of mathematical and engineering understanding in the answers on specific questions that included all the aspects.

The third research question deals with the interrelationships between the mathematical and engineering understanding, and Q4 of the questionnaire was designed to collect the students' perceptions of this issue. The results in Table 4 show that 6 students emphasized the similarity between the two, 17 students highlighted the differences between them, and 9 students stated that “mathematics is a basis/tool for engineering”. It is notable that more than one-third of students claimed that mathematics is a tool for engineering; yet at the same time, not even one student addressed the applicable aspect of mathematical understanding in the general question. This fact may stress again the students’ detached general perception of mathematical understanding as not applicable.

The current study deals with mathematical and engineering understanding. The significant learning of engineering takes place in academics of higher learning; therefore, it is reasonable to investigate engineering understanding among engineering students. Nevertheless, it is possible that initial perceptions about mathematical understanding are developed during high school studies. Therefore, it is recommended to conduct further research among high school students. Moreover, to expand the scope of the researched issue, it is suggested to address the same subject with a population of experts from the fields of mathematical and engineering education and compare the results with the current study.

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