A General and Theoretical FX Model for the Multi-Currency Basket: Economic, Financial and Mathematical Approach

Amine El Bied[1]

[1] l'École Nationale des Ponts et Chaussées ENPC, Paris

Correspondence: Amine El Bied, MBA, PhD, Ex-head of Dealing Rooms, Expert in Finance and Strategy, Economist; PhD in science from l'École Nationale des Ponts et Chaussées ENPC, Paris; MBA ESSEC Business School, Paris and Master's degree in Financial Markets ESSEC/CNAM.

Abstract

The multi-currency basket is an exchange rate regime in which the currency is pegged to several foreign currencies. The basket is defined by its composing foreign currencies, and by the weighting of each currency in the basket. The exchange rate in such a regime is pegged to a weighted basket of currencies.

We propose to present the benefits and the drawbacks of such an exchange rate regime from a macroeconomic and financial point of view. We also formulate mathematically a general theory of Foreign Exchange in the case of a currency under the regime of a multi-currency basket. The creation ex nihilo of a general model for a multi-currency basket results in equations applicable in all cases. The article covers theoretical and practical aspects of this exchange rate regime, responding to the most concrete issues faced by financial markets' professionals, and should also interest academics, teachers and students.

Keywords: multi-currency basket, exchange rate regime, fixed exchange rate, floating exchange rate, peg, pegged, foreign currencies, financial markets, central bank

1. Introduction

All the countries of the world have an exchange rate regime. It can be fixed or floating, or intermediate. The choice of an exchange rate regime is crucial for a country and can positively or negatively impact its economy. A country must therefore choose it well, and sometimes also decide to change it to adopt another regime more adapted to its economy. To make a relevant choice of exchange rate regime, it is necessary to know the advantages and disadvantages of each of them. We propose here to focus on the regime of a multi-currency basket. It's a fixed exchange rate regime pegged to several foreign currencies. The exchange rate in such a regime is fixed to a basket of weighted average currencies. We present here from a macroeconomic point of view the benefits and the drawbacks of such a multi-currency basket regime. We give thereafter a mathematical formulation of this exchange rate regime.

This mathematical formulation allows to master theoretically all the aspects of this exchange rate regime. It is meant to be very practical for central banks and traders intervening in the foreign exchange market, not to mention the interest it may have in academia, or for students in the financial and economic field.

The construction of a general model for a multi-currency basket results in equations applicable in all cases, whatever the currencies composing the basket, their nature, number and percentage in the basket. The model is therefore valid for all multi-currency baskets independently of their composition.

For this model, we start with a mathematical definition of a basket so that it's hedged against the variations of the currencies included in its composition. Then, we show the method to be used to determine in practice the percentage or the weight of each currency in the reference basket. We also show how to determine in practice the amount in each currency so that the portfolio composed of all the currencies of the basket has the value of one unit in the reference currency. We also show how to manage the exchange position, and how to determine the real-time quotes of exchange rates, in particular those of the reference currency with the currencies in the basket. We also discuss arbitrage between currencies in the basket, and with the base currency, as well as the cases of absence of arbitrage opportunities. We finally address the issue of exchange rate risk against the reference currency in the case of a long forward position in a currency, and the most adequate hedging using derivatives, where the other currency at stake is

the reference currency, or another currency entering into the composition of the basket. All these considerations will provide a better understanding of the exchange rate regime of a multi-currency basket, both theoretically and practically.

2. Benefits and Drawbacks of a Currency Basket Regime

The regime of a multi-currency basket is a fixed exchange rate regime. The exchange rate remains stable, reducing uncertainties and inflation risks. A fixed peg allows to fight hyperinflation. It gives confidence to investors and promotes an inflow of foreign capital and an increase in bank loans. But this exchange rate stability comes at a price. The value of the national currency is maintained artificially. This exchange rate regime requires the Central Bank to meet systematically the needs of banks in foreign currencies, to reduce pressure on these currencies and prevent the decline of the national currency. To support the national currency at a fixed exchange rate higher than the real value of the currency, one has to sell currencies, and therefore draw foreign exchange reserves, or borrow currencies, which will increase the debt. By supporting the national currency at a fixed exchange rate higher than its real value, a purchase in foreign currency is made cheaper for the domestic consumer, which means that the central bank subsidizes a part of the purchase. The fixed rate system is therefore similar to a compensation system, but which mainly benefits the wealthiest segment of the population, that buys foreign currencies and consumes foreign products, to the detriment of the rest of the population, including the poorest since the subsidy is paid by the State, and therefore by all the taxpayers. The fixed exchange rate regime is then a socially unfair system, a disguised State subsidy benefiting the richest while it should be concentrated on the poorest.

Supporting the national currency at a fixed exchange rate higher than its real value penalizes exporters and encourages imports. Foreign products are made more competitive and the competitiveness of national products is penalized, sometimes even making them more expensive, which affects local consumption. By favoring imports to the detriment of exports, a deficit in the trade balance is favoured too. This fixed exchange rate system makes the national economy vulnerable to external shocks and in times of crisis. An economic crisis or an exogenous shock, for example in the energy sector, will result in a rise in oil prices, and increase the foreign exchange needs of the oil-importing countries, which tends to devalue the national currency. To maintain a fixed rate, it will be necessary to draw abundantly foreign exchange reserves, which can dry them out. In such a fixed regime, the exchange regulations may be over-protectionist and limit the freedom of the economic operators. It has been said that the fixed peg gives confidence to investors and favors an inflow of foreign capital and an increase in bank loans, but bank lending can increase inflation to a high level, which will affect competitiveness in international trade and deteriorate the current account. Bank loans can also be used to finance real estate and financial speculations, and promote the creation of a speculative bubble, which can burst and lead to a fall in the real estate and stock markets and the withdrawal of capital. This fixed exchange rate system may become inappropriate to the economic context and the financial and social reality of the country. A currency basket regime may become unsuitable, for example in the event of a multiplication of free trade agreements signed with countries or regions whose national or regional currency is not one of the currencies included in the composition of the basket. The fixed exchange rate regime can be out of step with the need for a country to integrate into globalization and with its national ambition to become a financial platform, and more broadly a strategic place, in a region or a continent.

3. Mathematical Notations

Let n be the number of currencies in the basket of a reference currency "r".

Let C_{ij} or $C_{i/j}$ denote the rate of currency i against currency j (i and j: $1 \rightarrow n$). We have then:

$$1 \text{ i} = C_{i/j} \text{ j} = C_{ij} \text{ j}$$

Similarly: $C_{ir} = C_{i/r}$ with r the reference currency. We have: $1 \text{ i} = C_{i/r} \text{ r} = C_{ir} \text{ r}$

4. Formulas on Currency Rates

$1\text{i} = C_{ii}\text{i}$ and thus: $\mathbf{C_{ii}=1}$

$1 \text{ i} = C_{ij} \text{ j}$ then $1 \text{ j} = \frac{1}{C_{ij}}\text{i}$; and as $1 \text{ j} = C_{ji}\text{i}$ therefore: $\boldsymbol{C_{ji} = \frac{1}{c_{ij}}}$

$1 \text{ i} = C_{ir} \text{ r}$ and $1 \text{ r} = C_{rj} \text{ j}$ then $1 \text{ i} = C_{ir} \text{ r} = C_{ir} (C_{rj} \text{ j})$; and as $1 \text{ i} = C_{ij}\text{ j}$ therefore: $\boldsymbol{C_{ij} = C_{ir}C_{rj}}$

We obtain too: $\boldsymbol{C_{ir} = C_{ij}C_{jr}}$

5. Changes in Currency Rates

A variation of $C_{ij} \rightarrow C'_{ij}$ implies a variation of $C_{ir} \rightarrow C'_{ir}$ and of $C_{jr} \rightarrow C'_{jr}$ then: $\boldsymbol{C'_{ir} = C'_{ij} C'_{jr}}$

We denote: $\Delta \boldsymbol{C_{ij}} = \boldsymbol{C'_{ij} - C_{ij}}$

We have $\frac{\Delta C_{ij}}{C_{ij}} = \frac{C'_{ij} - C_{ij}}{C_{ij}}$; we replace C_{ij} by $\frac{C_{ir}}{C_{jr}}$ and C'_{ij} by $\frac{C'_{ir}}{C'_{jr}} = \frac{C_{ir} + \Delta C_{ir}}{C_{jr} + \Delta C_{jr}}$, we obtain then:

$$\frac{\Delta C_{ij}}{C_{ij}} = \frac{\frac{C_{ir} + \Delta C_{ir}}{C_{jr} + \Delta C_{jr}} - \frac{C_{ir}}{C_{jr}}}{\frac{C_{ir}}{C_{jr}}} = \left(\frac{C_{ir} + \Delta C_{ir}}{C_{jr} + \Delta C_{jr}} - \frac{C_{ir}}{C_{jr}} \right) \frac{C_{jr}}{C_{ir}} = \left(\frac{C_{ir} C_{jr} + C_{jr} \Delta C_{ir} - C_{ir} C_{jr} - C_{ir} \Delta C_{jr}}{C_{jr}(C_{jr} + \Delta C_{jr})} \right) \frac{C_{jr}}{C_{ir}} = \frac{C_{jr} \Delta C_{ir} - C_{ir} \Delta C_{jr}}{C_{jr}(C_{jr} + \Delta C_{jr})}$$

therefore: $\frac{\Delta C_{ij}}{C_{ij}} = \frac{C_{ir}}{C_{jr} + \Delta C_{jr}} \left[\frac{\Delta C_{ir}}{C_{ir}} - \frac{\Delta C_{jr}}{C_{jr}} \right]$

We have $C'_{ir} = C'_{ij} C'_{jr}$ then $C_{ir} + \Delta C_{ir} = (C_{ij} + \Delta C_{ij})(C_{jr} + \Delta C_{jr})$

$$\frac{C_{ij} + \Delta C_{ij}}{C_{ij}} = \frac{C_{ir} + \Delta C_{ir}}{C_{jr} + \Delta C_{jr}} / C_{ij} = \frac{C_{ir} + \Delta C_{ir}}{C_{jr} + \Delta C_{jr}} \frac{C_{jr}}{C_{ir}} = \frac{C_{ir} + \Delta C_{ir}}{C_{ir}} / \frac{C_{jr} + \Delta C_{jr}}{C_{jr}})$$

therefore: $\boldsymbol{1 + \frac{\Delta C_{ij}}{C_{ij}} = \frac{1 + \frac{\Delta C_{ir}}{C_{ir}}}{1 + \frac{\Delta C_{jr}}{C_{jr}}}}$

6. Definition of a Hedged Basket

Let's assume we have a nominal M_i in the currency i, namely M_i (i). Its equivalent in the reference currency r is $M_i C_{ir}$ (r). We have n currencies: i: $1 \rightarrow$ n; and a nominal M_i for each currency i. Its equivalent in the reference currency r is $\sum_{i=1}^{n} M_i C_{ir}$ (r)

A basket in the currency r is hedged if any variation in relation to each other of the rates of the n currencies of the basket, in other words any variation in C_{ij} (i and j: $1 \rightarrow$ n), has not impact on the valuation of this basket in the currency r.

The portfolio consisted of amounts M_i in each currency i of the basket keeps an unchanged value in currency r regardless of the evolution of the rates C_{ir}:

$$\sum_{i=1}^{n} M_i C_{ir} \text{ (r)} = \text{constant (time-independent)} = M \text{ (r)}$$

$C_{ir} \rightarrow C'_{ir}$ and M_i with i: $1 \rightarrow$ n such as: $\sum_{i=1}^{n} M_i C'_{ir}$ (r) $= \sum_{i=1}^{n} M_i C_{ir}$ (r) = constant, and then:

$$\sum_{i=1}^{n} M_i \Delta C_{ir} = 0$$

7. Share of Currencies in a Hedged Basket

$$\frac{\sum_{i=1}^{n} M_i C_{ir}}{\sum_{i=1}^{n} M_i C_{ir}} = 1$$

and then:

$$\underbrace{\frac{M_1 C_{1r}}{\sum_{i=1}^{n} M_i C_{ir}}}_{\alpha_1 \%} + \ldots + \underbrace{\frac{M_i C_{ir}}{\sum_{i=1}^{n} M_i C_{ir}}}_{\alpha_i \%} + \ldots + \underbrace{\frac{M_n C_{nr}}{\sum_{i=1}^{n} M_i C_{ir}}}_{\alpha_n \%} = 1 = 100\%$$

so: $\boldsymbol{\alpha_i \% = \frac{M_i C_{ir}}{\sum_{i=1}^{n} M_i C_{ir}} = \frac{M_i C_{ir}}{M \text{ (r)}}}$

$\alpha_i\%$ represents the share or the weight of the currency i in the reference basket r.

We have then: $\sum_{i=1}^{n} \boldsymbol{\alpha_i}\% = \mathbf{1}$

We have: $\alpha_j\% = \frac{M_j\, C_{jr}}{M\,(r)} = \frac{M_j\, C_{jr}}{M\,(r)}$, therefore: $\frac{\boldsymbol{\alpha_i}\%}{\boldsymbol{\alpha_j}\%} = \frac{\mathbf{M_i C_{ir}}}{\mathbf{M_j C_{jr}}}$ or $\frac{\alpha_i\%}{M_i\, C_{ir}} = \frac{\alpha_j\%}{M_j C_{jr}}$

or: $M_i\, C_{ir} = (\frac{\alpha_i\%}{\alpha_j\%})\ M_j\, C_{jr}$

This means that M_i is such that the quantity in currency r from the counter value of currency i represents $\alpha_i\%$, and therefore $(\frac{\alpha_i\%}{\alpha_j\%})$ times the quantity in r from the counter value j, which represents $\alpha_j\%$.

There is a proportion of $\alpha_i\%$ - $\alpha_j\%$ or $\frac{\alpha_i\%}{\alpha_j\%}$ - 1 between the two currencies i and j.

Still in the general case of n currencies, we look for $\alpha_i\%$ such that the basket is hedged, and therefore such that $\sum_{i=1}^{n} M_i\, C_{ir}\,(r) = $ constant.

We have: $\alpha_i\% = \frac{M_i\, C_{ir}}{\sum_{i=1}^{n} M_i\, C_{ir}}$

with M_i constant, $\sum_{i=1}^{n} M_i\, C_{ir}$ constant, but C_{ir} is not constant, so $\alpha_i\%$ is not a constant either.

$\alpha_i\%$ is equal to C_{ir} multiplied by a constant.

We denote: $\boldsymbol{\beta_i}\% = \frac{\mathbf{M_i}}{\sum_{i=1}^{n} \mathbf{M_i C_{ir}}}$

$\beta_i\%$ is a constant and: $\boldsymbol{\alpha_i}\% = \boldsymbol{\beta_i}\%\ \mathbf{C_{ir}}$

We have $\sum_{i=1}^{n} \alpha_i\% = 1$

and therefore: $\sum_{i=1}^{n} \boldsymbol{\beta_i}\% \,\mathbf{C_{ir}} = \mathbf{1}$

$\beta_i\%$ represents the amount in the currency i so that the portfolio consisted of all the currencies of the basket r has a value of one (1) in the currency r.

As $\sum_{i=1}^{n} M_i\, C_{ir}\,(r) = $ constant, then $\sum_{i=1}^{n} M_i\, C_{ir}\,(r) = \sum_{i=1}^{n} M_i\, C'_{ir}\,(r)$

By replacing in the following equation:

$$\frac{\sum_{i=1}^{n} M_i\, C'_{ir}}{\sum_{i=1}^{n} M_i\, C'_{ir}} = 1 = \frac{\sum_{i=1}^{n} M_i\, C'_{ir}}{\sum_{i=1}^{n} M_i\, C_{ir}} = \frac{M_1\, C'_{1r}}{\sum_{i=1}^{n} M_i\, C_{ir}} + \ldots + \frac{M_i C'_{ir}}{\sum_{i=1}^{n} M_i\, C_{ir}} + \ldots + \frac{M_n\, C'_{nr}}{\sum_{i=1}^{n} M_i\, C_{ir}}$$

$$1 = \beta_1\%\, C'_{1r} + \ldots + \beta_i\%\, C'_{ir} + \ldots + \beta_n\%\, C'_{nr}$$

therefore: $\sum_{i=1}^{n} \boldsymbol{\beta_i}\% \,\boldsymbol{C'_{ir}} = \mathbf{1}$

therefore: $\sum_{i=1}^{n} \boldsymbol{\beta_i}\% \,\Delta\mathbf{C_{ir}} = \mathbf{0}$

8. Determination of $\boldsymbol{\beta_i}\%$ and $\boldsymbol{\alpha_i}\%$ in Practice

We would like to calculate $\alpha_i\%$ for each currency i (i: $1 \rightarrow$ n), $\alpha_i\%$ representing the share of currency i in the basket of reference currency r.

We start by determining the $\beta_i\%$ (which are constants). We're looking for $\beta_i\%$ such that:

$$\sum_{i=1}^{n} \beta_i\%\, C_{ir} = 1$$

In practice, the $\beta_i\%$ are determined on the basis of historic exchange rates from a linear regression analysis (of a column of 1 denoted Y in relation to C_{ir}) or by the method of least squares (by minimizing (Y - $\beta_i\%\ C_{ir}$)^2).

These $\beta_i\%$ do not change from one date to another. These are constants determined by regression.

On the other hand, from one date to another the C_{ir} (for i: 1 → n) change, and then the composition

$\alpha_i\% = \beta_i\%\ C_{ir}$ of each currency i in the basket changes.

To obtain an $\alpha_i\% <0$ does it make sense?

$\alpha_i\% < 0$ is equivalent to $\frac{M_i\ C_{ir}}{\sum_{i=1}^{n} M_i\ C_{ir}} < 0$, thus $M_i\ C_{ir} < 0$ and therefore $M_i < 0$. This means that in the composition of the basket hedged in the currency r, we have M_i in the currency i with $M_i < 0$, and therefore we are short on this currency.

If we don't know the composition of a basket and we try to determine it, the difficulty also lies in the choice of the currencies on which the calculation will be made. Logically, if $\alpha_i\%$ is obtained very close to zero or even insignificant for a given currency i, this tends to show that this currency is not included in the basket. Another difficulty lies in the choice of the historical data on which the calculations will be based, in terms of the period chosen and the frequency of the data (daily or other time intervals).

9. The Foreign Currency Management

We have: $\sum_{i=1}^{n} \beta_i\%\ C_{ir} = 1$

$\beta_i\%$ represents the amount in the currency i so that the portfolio made up of all the currencies of the basket r has the value one (1) in the currency r.

We have: $\sum_{i=1}^{n} M_i\ C_{ir}\ (r) = \text{constant} = M\ (r)$

So M_i represents the amount in the currency i so that the portfolio composed of all the currencies of the basket r has the value M in the currency r. It's up to us to choose the amount M we want, representing the value of the portfolio in the currency r. We then deduce the amounts M_i .

We have: $\sum_{i=1}^{n} \beta_i\%\ C_{ir} = 1$

We multiply by M(r) on both sides, knowing that M (r) is a constant,

Then $\sum_{i=1}^{n} \beta_i\%\ M\ (r)C_{ir} = M\ (r)$ and as $\sum_{i=1}^{n} M_i\ C_{ir}\ (r) = M\ (r)$

Then $M_i = \beta_i\%\ M\ (r)$ and as $\alpha_i\% = \beta_i\%\ C_{ir}$ thus:

$$M_i = M\ (r)\ \frac{\alpha_i\%}{C_{ir}}$$

We have determined the target positions (long if $M_i > 0$) in the currencies i for i: 1 → n such that the basket r is hedged.

10. The Real-Time Quotes

We are here in the case where the rates between the currencies i and j (i and j: 1 → n) are available in real time, while the rates involving the currency r are displayed only with some delay by the central bank of the currency concerned. If the following rates are displayed:

The rates at a date t-1: $C_{ir\ (t-1)}$ (i: 1 → n) and $C_{ij\ (t-1)}$ (i et j: 1 → n);

and the rates $C_{ij\ (t)}$ (i et j: 1 → n). How to calculate from it the rate $C_{ir\ (t)}$ (i: 1 → n)?

We have: $\sum_{i=1}^{n} \alpha_i\% = \sum_{i=1}^{n} \beta_i\%\ C_{ir} = 1$

and as $C_{ir} = C_{ij}\ C_{jr}$

so by replacing:

$$\sum_{i=1}^{n} \beta_i\% \, C_{ir} = \sum_{i=1}^{n} \beta_i\% \, C_{ij} \, C_{jr} = C_{jr} \sum_{i=1}^{n} \beta_i\% \, C_{ij} = 1$$

and then: $C_{jr} = \dfrac{1}{\sum_{i=1}^{n} \beta_i\% \, C_{ij}}$

or: $C_{rj} = \sum_{i=1}^{n}(\beta_i\% \, C_{ij})$

or: $C_{rj} = \beta_j\% + \sum_{i=1; i\neq j}^{n}(\beta_i\% \, C_{ij})$

So this is the relation between C_{jr} and C_{ij} (i: $1 \rightarrow$ n). If we have these C_{ij}, we can deduce the rates involving the reference currency, using the values of $\beta_i\%$ already estimated.

In a practical case, if we do a linear regression of C_{rj} on the rates C_{ij} (with i: $1 \rightarrow$ n), we can thus determine the values of $\beta_i\%$ (with i: $1 \rightarrow$ n), and therefore by multiplying by C_{ir}, we obtain the shares $\alpha_i\%$ of the currencies i in the basket r. Furthermore, by replacing $\beta_i\%$ by $\alpha_i\% \, C_{ri}$ in the previous formula, we obtain:

$$C_{rj} = \alpha_j\% C_{rj} + \sum_{i=1; i\neq j}^{n}(\alpha_i\% C_{ri} C_{ij})$$

and as: 1 r = C_{rj} j

then: $\mathbf{1\ r = \{\alpha_j\% C_{rj} + \sum_{i=1; i\neq j}^{n}(\alpha_i\% C_{ri} C_{ij})\}j}$

This equation can be seen as an alternate formulation of a basket r.

11. Arbitrage Involving Currency i / Currency j

Over n days, we borrow M_j at the rate $\theta_{j\,ask}\%$, we buy i / j at $C_{ij\,ask}$ and we loan M_i at the rate $\theta_{i\,bid}\%$.

After n days, we recover the money in the currency i, we sell i/j at $C_{ij\,bid}$ and we repay the loan in the currency j. In the end:

$$\mathbf{P\&L = M_j[\frac{C_{ijbid}}{C_{ijask}}\ (1 + \theta_{ibid}\% \ast n/360) - 1 - \theta_{jask}\% \ast n/360\ ]}$$

M_j is the initial amount borrowed. What's in the square brackets [ ] is a very small amount, which is generally negative. If so, there is No Arbitrage Opportunity.

12. Arbitrage Involving the Currency r

Over n days, we borrow M (r) at the rate $\theta_r\%$. We buy M_i i for each currency i:$1\rightarrow$ n to have a hedged basket r, and therefore we can use the following formula:

$M_i = M (r) \dfrac{\alpha_i\%}{C_{ir\,ask}}$ (we buy the currencies at an ask rate)

then we have: $\sum_{i=1}^{n} M_i \, C_{ir\,ask} (r) = $ constant $= M (r)$

We lend M_i i at the rate $\theta_{i\,bid}\%$ for each currency i: $1\rightarrow$ n.

After n days, we recover the investment in the currency i:

$M_i (1 + \theta_{i\,bid}\% \ast n/360)$ i for each currency i.

We sell i/r at C'_{irbid}, we obtain: $\sum_{i=1}^{n} M_i \left(1 + \theta_{i\,bid}\% \ast \dfrac{n}{360}\right) C'_{irbid} (r)$

By replacing $M_i = M (r) \dfrac{\alpha_i\%}{C_{ir\,ask}}$, we obtain:

$$\sum_{i=1}^{n} M\,(r)\,\frac{\alpha_i\%}{C_{ir\,ask}}\left(1\,+\,\theta_{i\,bid}\,\%\,*\frac{n}{360}\right)C'_{irbid}\,(r)$$

As we repay $M\,(r)\,(1\,+\,\theta_{r\,ask}\%)$, we have in the end:

$$\mathbf{P\&L} = \,\mathbf{M}\,(r)\,\left[\,\left(\sum_{i=1}^{n}\frac{C'_{irbid}}{C_{irask}}\boldsymbol{\alpha_i}\%\left(1\,+\,\boldsymbol{\theta_{ibid}}\%\,*\frac{n}{360}\right)\right)-1-\,\boldsymbol{\theta_{rask}}\%\,\right]\,(\mathbf{r})$$

$M(r)$ is the initial amount borrowed. What's between the [ ] is a very small amount.

It is worth having a foreign currency position based on the composition of the basket r to take advantage of interest rate spreads between the currency r and the currencies i (: $1 \rightarrow n$) if these rate spreads are such that P&L > 0, or

$$\sum_{i=1}^{n}\frac{C'_{irbid}}{C_{ir\,ask}}\boldsymbol{\alpha_i}\%\left(\mathbf{1}\,+\,\boldsymbol{\theta_{i\,bid}}\,\%\,*\frac{\mathbf{n}}{\mathbf{360}}\right) > (1 + \boldsymbol{\theta_{r\,ask}}\,\%)\,(\mathbf{r})$$

It will be in our interest to keep the position. This is particularly the case if we do not need to borrow the entire amount $M\,(r)$, having for example equity on which we do not pay interest. In the case where the above equation does not hold, there is No Arbitrage Opportunity.

13. Hedging of a Foreign Exchange Risk j/r by Derivatives i/j

A forward long position in a currency j, in x months, constitutes a foreign exchange risk in the reference currency r. The most adequate hedge for a foreign exchange risk currency j / currency r is the use of j / r derivatives. But in the event of a limited liquidity in this derivatives market, in terms of number of potential counterparties and the variety of products, it is possible to hedge with derivatives i / j (i: $1 \rightarrow n$ and j one of these currencies). The currencies i are included in the basket of the reference currency r, and the derivatives i / j are for example forward buying ij (forward selling ji) or buying a put ji.

Let $C_{ir\,x\,mois}$ be the spot price i/r in x months. Assuming that r is made up of a basket of currencies (i: $1 \rightarrow n$). We are for example long in a currency j in x months of M'_j j (there is concern of a decrease in j / r in x months).

To hedge a long position in j, we can make several forward purchases i / j (or forward sales j / i) in x months with i: $1 \rightarrow n$, $i \neq j$, at a price F_{ij} (we price it and we ask for quotation) on a nominal value of M_i i, in order to reach the composition of the basket in the reference currency r.

In x months, the value of M'_j j is $(M'_j C_{jr\,x\,mois})\,(r)$

For i: $1 \rightarrow n$, $i \neq j$, we buy M_i i against $(M_i\,F_{ij\,x\,mois})$ j such as:

M_i i is equivalent to $\alpha_i\%\,(M'_j C_{jr\,x\,mois})$ r, so:

$(M_i\,C_{ir\,x\,mois})\,r = \alpha_i\%\,(M'_j C_{jr\,x\,mois})$ r or $M_i = \alpha_i\%\,M'_j\frac{C_{jr\,x\,mois}}{C_{ir\,x\,mois}}$ so:

$$\mathbf{M_i} = (\boldsymbol{\alpha_i}\%\frac{\mathbf{M'_j}}{\mathbf{C_{ij\,x\,mois}}}\,)\,\mathbf{i}\ (\text{for i: } 1 \rightarrow n,\ i \neq j)$$

Today, $C_{ij\,x\,mois}$ is estimated with F_{ij} (we don't know $C_{ij\,x\,mois}$, but we can calculate F_{ij} from $C_{ij\,t=0}$). We can then calculate M_i, the nominal value on which we make the forward purchase, therefore:

$$\mathbf{M_i} = (\boldsymbol{\alpha_i}\%\frac{\mathbf{M'_j}}{\mathbf{F_{ijxmois}}})\,\mathbf{i}$$

We still have in the currency j:

$$\mathbf{M_j} = (\mathbf{M'_j}\,-\,\sum_{i=1\,i\,\neq j}^{n}\mathbf{M_i\,F_{ijxmois}})\,\mathbf{j}$$

This amount must correspond to $(\alpha_j\%\,M'_j C_{jr\,x\,mois})$ r

(The amount remaining in j must also correspond to the composition of the basket r and therefore be in proportion $\alpha_j\%$), and therefore:

$$[(M'_j - \sum_{i=1\ i \neq j}^{n} M_i F_{ij})\ C_{jr\ x\ mois}]\ r = \alpha_j\% (M'_j C_{jr\ x\ mois})\ r$$

so: $M_j = (M'_j - \sum_{i=1\ i \neq j}^{n} M_i F_{ij}) = \alpha_j\% M'_j$

$$\sum_{i=1\ i \neq j}^{n} M_i F_{ij} = M'_j\ (1 - \alpha_j\%)$$

By replacing M_i: $\sum_{i=1\ i \neq j}^{n} (\alpha_i\% \frac{M'_j}{C_{ij\ x\ mois}}) F_{ij} = M'_j\ (1 - \alpha_j\%)$

We simplify by M'_j and we estimate that $C_{ij\ x\ mois} = F_{ij}$, we obtain then: $\sum_{i=1\ i \neq j}^{n} \alpha_i\% = 1 - \alpha_j\%$,

so: $\alpha_j\% + \sum_{i=1\ i \neq j}^{n} \alpha_i\% = 1$, so: $\sum_{i=1}^{n} \alpha_i\% = 1$.

It does, so it makes sense.

If we therefore make forward purchases i/j in x months of M_i i at a price F_{ij}, such as

$M_i = (\alpha_i\% \frac{M'_j}{C_{ij\ x\ mois}})$ i, for i: $1 \rightarrow$ n, i $\neq$j), this hedge is good if we assume that the basket in r is composed of a share

$\alpha_i\%$ for each currency i (i: $1 \rightarrow$ n). We hedge the evolution of j/r by derivatives i/j (i: $1 \rightarrow$ n, i $\neq$j).

14. Conclusion

We presented from a macroeconomic point of view the benefits and the drawbacks of an exchange rate regime of a multi-currency basket. We proposed thereafter a general mathematical formulation of a multi-currency basket regime. After having mathematically defined a basket hedged against the variations of the currencies composing it, we have shown in a practical way how to determine these currencies and their weight in the basket. For a basket value of one unit in the reference currency, we have shown how to determine the amount of each currency included in the basket. We have shown how the exchange position should be managed, and how to determine in a practical way the real-time quotes of exchange rates, and more particularly the exchange rates of the reference currency against the currencies included in the composition of its basket. We then showed the method and the formulas to be used to know if there are no arbitrage opportunities or if there is a possibility of arbitrage between two currencies of the basket or between a currency of the basket and the reference currency. We have shown finally how to hedge the exchange risk, against the reference currency, of a long forward position in a currency of the basket, by using derivatives where the other currency concerned can be either the reference currency, or another currency in the basket.

The exchange rate regime of a multi-currency basket has been addressed in this paper in all its most practical and theoretical aspects, by a financial approach, and a mathematical formulation which leads to a general model, valid and applicable in all cases, whatever the number of currencies, their nature and their weightings in the basket.

This paper is about the regime of a multi-currency basket, but it is possible to extend this research work to other currency exchange regimes, once again analyzing the advantages and disadvantages for each of them, to allow different countries to choose an exchange rate regime that is more adapted to their economy. We could also give the mathematical formulation to define the theoretical outlines, which would again have a very practical interest.

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