An Extended Stochastic Goal Mixed Integer Programming for Optimal Portfolio Selection in the Amman Stock Exchange

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Abstract

This paper optimally solves the portfolio selection problem that consists of multi assets in a continuous time period to achieve the optimal trade-off between multi-objectives. In this paper, the Stochastic Goal Mixed Integer programming of Stoyan (2009) is extended. The empirical contributions of this research presented on extending the SGMIP model by adding information as a new factor that selects the portfolio elements. The information element used as a portfolio managing characteristics to see whether it is applicable for different problems. The data was collected on a daily basis for all the parameters of the individual stock. Brownian motion formula was used to predict the stock price in the future time period. SP framework used to capture numerous sources of uncertainty and to formulate the portfolio problem. The main challenge of this model is that it contains additional real-world objective and multi types of financial assets, which form a Mixed Integer Programming (MIP). This large-scale problem solved using Optimising Programming Language (OPL) and decomposition algorithm to improve the memory allocation and CPU time. A fascinating result was obtained from the portfolio algorithm design. The ESGMIP portfolio outperforms the Index portfolio return. Under uncertain environment, the availability of information rationalized the diversity when the dynamic portfolio invested in one financial instrument (stocks), and tend to be diversifiable when invested in more than one financial instrument (stock and bond). This work presents a novel extended SGMIP model to reach an optimal solution.

Keywords: Portfolio Selection (PS), SMIP, SGMIP

1. Introduction

Modern Portfolio Theory has emerged many different models sought to provide some assistant in decision making environment. Each model is a simplification or simulation of reality (Pástor, 2000). By capturing the real world features, models become more complex, therefore many attempts provided as a simplification. Researchers approached portfolio selection differently; some of them approach mean-variance of Markowitz (1952) focusing on the trade-off between risk and return neglecting other essential factors. Therefore, all models aimed to maximize the return attached with specified level of risk assuming that it will satisfy the investors' interests. The wide applications of the models were neither desirable nor important (Azmi and Tamiz, 2010). Accordingly, the portfolio selection problem gets enlarged and remains unsolved, even after extending to involve other factors such as liquidity, cardinality constraint, transaction cost, short sale and etc., which encourage the researchers to apply either other risk measures or simplify the mathematical models.

The problem of portfolio selection is in the scarce of resources. It is not just which stocks to own, but how to distribute the investor's wealth amongst stocks (Ravipati, 2012). In Finance, portfolio selection is famous as a leading problem; giving that the future return of an asset is unknown when investment decision made therefore, the decision making is under uncertainty: one can evaluate the today decision just in future time once the assets return is revealed (Roman and Mirta, 2008). Stochastic approach is a common technique in portfolio decision-making to create a model for random uncertainty conditions in the financial markets. It deals with the uncertainties in a flexible way and incorporates real-world constraints easily (Yano, 2014).

Optimising the dynamic portfolio depends significantly on the variation of stock price. At the same time, it is difficult to rely on it in estimating the variation and volatility parameters for future periods (Frey, Gabih & Wunderlich, 2013). In this situation, it is necessary to use the sequence action model under uncertainty introduced by Valian (2009). These actions were measured based on estimated drift using Browian motion (Al-Halaseh, 2016). Considering the uncertainty in stock price, the portfolio parameters measured as stochastic variables.

Acknowledging the above mentioned facts, the most recent method, which is the Stochastic Goal Mixed Integer Programming model, has been applied to Toronto Stock Exchange (TSX) in 2009. Until today, it is the most proper and effective model due to its compatibility of including a number of financial instruments with real-world application (Al-Halaseh, 2016). It considers multi-factors, multi-periods, different risk measures without affecting time which facilitates the mission of decision makers. However, Stoyan (2009) acknowledged that the algorithm was pushed to its limits with regards to the number of variables and solvability. In line with this, additional constraints and scenarios would present an even larger problem for algorithm design and solvability. Thus, the study recommended expanding the size of the problem by adding constraints and scenarios. It would mean that the algorithm would also have to be tailored to facilitate such additions. It should also be applied to different markets to verify the results. Therefore, there is a need to extend the existing SGMIP in order to capture more factors that would make it more effective in selecting an optimal portfolio.

Based on the literature review, it is observed that there are relevant factors that can be used to optimize dynamic portfolio. One of which is the quality of information that flows into the market. The fluctuation faced by the ASE index is contributed by the nature of the investors (individual investors) of ASE who focus on speculation rather than on the usual investment objectives to make capital gains, and the weak role of the institutional investors, which open the way for rumours to grow (Al-Emam, 2010). Therefore, there is a need to fill the gap by extending the SGMIP model with the inclusion of the information factor to see whether the DP selection can be effectively optimised. Moreover, there is a need to expand the application of the extended model to other markets especially in ASE since no attempt has been made in ASE to find out the most effective model in optimizing the dynamic portfolio selection. Thus, this creates a motive to the researcher to make a significant addition to portfolio optimization, facilitating the investors in selecting their optimal portfolio, satisfy their investment goals, and at the same time, investigate the applicability of the new model in markets such as ASE.

Moreover, Stochastic Goal Mixed-Integer Programming helps to answer the questions that have emerged in recent year, such as “How to optimally allocate and arrange the investor’s available resources at each period during a time horizon in an uncertain environment?”, “To what extent are the investors able to take an optimal decision to minimize portfolio risks that are subjected to satisfying the other objectives of minimizing cardinality constraint, transaction cost, and amount of each security in order to maximise diversity, liquidity, accuracy of available information and portfolio return?”, “How can real-world dynamic (multi-period) portfolio selection be efficiently solved by using SGMIP with samples from ASE?”. These questions, along with many others, need to be answered in a developing market context as the original model used fewer constraints and has been applied to a small market such as TSX when tested from the context of the developed market. Therefore, the study will attempt to apply the extended SGMIP on a portfolio that includes all listed companies and bonds in ASE during the period from January 2010 to December 2014.

Due to the present economic condition of Jordan, historical price movements, investment trends, and stock-bond relationship, this research studies pure stock portfolio and stock-bond portfolio selection. Additionally, the uncertainty linked with the current financial market is captured in a fundamental SMIP approach with recourse. Previous studies (Ibrahim, 2008; Stoyan, 2009; Stoyan & Kwon, 2010, 2011) documented that long-term investment is one of the best financial strategies to maximise expected returns. Therefore this study has attempted to develop a long-term portfolio strategy, while considering both pure stock and stock-bond investment to capture the risk taker and risk adverse perspective respectively. Such portfolios are difficult to construct and are computationally demanding. This study faced a trade-off between trying to capture numerous realistic portfolio elements and solving the optimality problem. Issues such as the number of time-stages, type of portfolio elements, number of portfolio elements, and abundant uncertainties involved in developing long term financial strategies require greater consideration. Consequently, this research has been conducted to achieve its objectives by answering the research questions.

The remaining parts of this paper are organized as follows. Section two discusses the ESGMIP model, while section three discusses model implementation; Section four revealed the results of ESGMIP model; Section five displayed the portfolio discussion and further studies.

2. The ESGMIP Model

Two types of decision variables related to the assets are included in the portfolio, which are stocks and bonds. Thereafter, the portfolio elements and the ESGMIP will be formulated in specific. The critical portfolio elements the study model will consider are as per the following:

1. Minimise the portfolio risk
2. Hold a small number of investment
3. Minimise transaction cost
4. Maximise liquidity
5. Maximise the quality (accuracy) of information.
6. Maximise the portfolio return
7. Achieve diversity by minimising unsystematic risk
8. Stay within a tolerance of market performance measure

The explanation for including the portfolio elements (2), (3), (4) and (5) is to avoid facing the liquidity issue especially this study is investing in the long-term portfolio and may some stocks or bonds have a small number of issuing shares. Consequently, this study managing dynamic portfolio for that rebalancing is very essential for it, thusly, transaction cost, liquidity; the number of assets and availability of information are in favour of events. Besides, the number of assets is incorporated into all portfolio designs and has a great practical value which is considered as a necessity by the portfolio manager (Stoyan, 2009; He & Qu, 2014). In this way, these components shape a multi-objective SMIP. Then again, the GP approach is utilized to fulfil these objectives. Additionally, every one of these components are captured in the portfolio selection model will be characterized utilizing the two specified portfolios.

2.1 The Portfolio Selection Formulation

In this section, the elements considered within the portfolio model are presented so as to get the final ESGMIP portfolio design. Multi-period PSP is investigated with a set of real-world constraints under uncertainty condition. After assessing the model of Stoyan (2009) in the ASE, it appears that this model needs to be expanded in order to include most of the conditions facing ASE which imposed adding an extra constraint, i.e. information availability and the amount of individual asset. Obtaining the portfolio problem requires defining the portfolio elements that consist of transaction cost, liquidity, diversity by minimizing the unsystematic risk, risk and return and their vectors referred to stocks and bonds. Including these elements to the portfolio of this study is because they suit the long-term strategy and the stock-bond investment. The variables of the model define as, x_i, is the fraction of the portfolio invested in security *i* that is purchased in the first-stage (t=0). y_{il}^t is the fraction of the portfolio invested in security *i* that is purchased in the second-stage (t>0). $Ø_{il}^t$ is the unit price of security *i* at time t = 0,1,…,m under scenario l = 1,2,… L , i = 1,2,…n .Where $x_i \in R$, and $y_{il}^t \in R$, note that the security price is known at t = 0 and there is only one scenario in the first stage. z_{jl}^t is the fraction of the portfolio invested in bond *j* to purchase at time *t* under scenario *l*, hence $z_{jl}^t \in R$. φ_{jl}^t is the price of bond *j* at time *t* under scenario *l*. U_{jl}^t is the bond return at maturity. $\hat{B}$ is the initial wealth of the portfolio. In order to define the portfolio elements, this paper begin by maximising the return of the portfolio as in the following equation:

$$\sum_{l=1}^{L}\sum_{i=1}^{n} Ø_{il}^{1} x_i + \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{i=1}^{n} pl\, Ø_{il}^{t+1}\, y_{il}^{t} + \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{j=1}^{h} pl\, U_{jl}^{t}\ z_{jl}^{t-h_j^*} \tag{1}$$

Where p_l denotes the probability of a scenario realisation, where $\sum_{l=1}^{L} pl = 1$ and pl > 0. The model aims to maintain the minimum level of portfolio managing fees, which entails minimising transaction costs and therefore minimising the number of transactions between time periods. Thus, defining transaction cost $\ddot{w}_{il}^{t}$ to be the following:

$$\varpi_{il}^t = |\, y_{il}^t - y_{il}^{t-1} |\ i= 1,2,\ldots,n;\ t=2,\ldots,T;\ l = 1,\ldots,L \tag{2}$$

for t=1

$$\varpi_{il}^1 = |\, y_{il}^t - x_i |\ \ i=1,2,\ldots,n;\ \ l=1,\ldots,L \tag{3}$$

Where ϖ_{il}^0 = 0, ϖ_{il}^t equals the fraction of a security that traded between two periods. The previous equation will be minimised in objective function in order to maintain the portfolio cost to a minimum. The second objective is to maximise sector diversity and minimise the portfolio risk. The risk (unsystematic risk) will be reduced by diversifying the portfolio throughout the market sectors (sector diversified). Therefore, diversification is a guarantee for poor sector development which can be a result of a number of securities. To include the sector exposure element, the variable Q(i, s) determine the security i to which sector is belonged; Hence, Q(i, s) = 1, if security i belonged to sector s ,otherwise = 0, where S represents the total of sectors, Q(i, s) $\in$ B. To give the portfolio the proper sector diversification, let's consider f_s^t is the fraction of the portfolio, which will be as follows

$$\sum_{s=1}^{S} f_s^t = 1;\ t = 0, \dots, T \tag{4}$$

$f_s^{\,t} \in [0, 1]$, knowing that $f_s^{\,0}$ is a first stage parameter, and $f_s^{\,t}$ is a second stage parameter when t> 0. The equation (5) forms the sector exposure element.

$$\sum_{i=1}^{n} \mathrm{Q}(i, \mathrm{s})\, \text{Ø}_{il}^{t} \mathrm{y}_{il}^{t} = f_s^t \sum_{i=1}^{n} \text{Ø}_{\mathrm{il}}^{\mathrm{t}}\ \mathrm{y}_{\mathrm{il}}^{\mathrm{t}} + \xi_{\mathrm{sl}}^{\mathrm{t}} \tag{5}$$

Where $\xi_{\mathrm{sl}}^{\mathrm{t}}$ is a sector penalty variable that compatibles to the fraction of the portfolio f_s^t that invested in sector s. Sector penalty variable assists the model to find other fraction of the portfolio if the feasible solution cannot be found with the current used variable. The above constraint can be used for variable x_i by replacing the variable y_{il}^t.

To limit the number of divers securities and bonds utilised as a part of the portfolio, assume g_{il}^t is quantity of securities i invested in the portfolio at time t under scenario l , $\mathrm{g}_{il}^t \in B$,where there is one scenario in the first stage l= 1 for g_{il}^0, as follows:

$$\mathrm{g}_{il}^{t} = \begin{cases} 1, \text{ if security } i \text{ is used in the portfolio at time } t \\ \text{under scenario } l \text{ (i.e. if } xi,\ y_i^t > 0); \\ 0, \text{ otherwise} \end{cases} \tag{6}$$

Therefore considering G^t as the upper limit of the quantity of stocks to hold in the portfolio and in order to achieve the goal of limiting number (quantity) of security to hold the cardinality constraint will be:

$$\sum_{i=1}^{n} \mathrm{g}_{il}^t \leq \mathrm{G}^{\mathrm{t}}\ ;\ t = 0,\dots,\ \mathrm{T},\ l = 1,\dots,\mathrm{L}. \tag{7}$$

To ensure managing the portfolio to invest with sufficient funds, let's $\hat{B}$ represents the initial wealth of the portfolio. Thusly, the constraints of balancing the portfolio are as follows:

$$\hat{B} = \sum_{i=1}^{n} \text{Ø}_i^0 x_i + \sum_{j=1}^{h} \varphi_{jl}^0 z_j^0 \tag{8}$$

$$B_l^1 = \sum_{i=1}^{n} \text{Ø}_{il}^1 x_i - \sum_{i=1}^{n} \text{Ø}_{il}^1 y_{il}^1 + \sum_{j=1}^{h} U_{jl}^1 Z_j^{1-h_j} - \sum_{j=1}^{h} \varphi_{jl}^1 Z_j^1 - \tau_l^1 \overline{w}_{il}^1 \tag{9}$$

Where, τ is the relative cost of a security transaction. Thus, (10)-(11) ensures that all portfolio wealth is being invested at each time period, including dividends. Additionally, upper bounds are added to security and bond decision variables to further force diversity, where $\mathrm{d_i}$ and $\tilde{\mathrm{d}}_{\mathrm{j}}$ are the maximum fractions of the portfolio to be invested in security i or bond j; respectively. Next, this paper introduces the various portfolio goals the model will account for by adding the GP approach to the problem.

2.2 The ESGMIP Model Design

The previous section displays some elements included in the portfolio. This segment displays the rest of the components as targets objectives or goals which are risk, return and liquidity. These objectives tackled in particular portfolio problems to acquire the optimal value then constraint them as goal constraints. Any deviation from the optimal value will penalize in the objective function. Showing the performance measure as the primary portfolio objective is to guarantee that the portfolio can't beat the acquired optimal value. To do as such, suppose R_l^t is a maximum benchmark the investment not permitted to beat at time t under scenario l. The estimation of R_l^t is acquired after calculating the index return. The performance constraint is as per the following:

$$\text{For first stage } \sum_{i=1}^{n} \emptyset_{il}^{t+1} x_{il}^{t} \leq R_l^t + \mathrm{X}_l^{\mathrm{t}};\ t = 1, l = 1 \tag{10}$$

$$\text{For first stage } \sum_{i=1}^{n} \emptyset_{il}^{t+1} y_{il}^{t} \leq R_l^t + \mathrm{X}_l^{\mathrm{t}};\ t = 1, \dots \mathrm{T}, l = 1$$

Where $\mathrm{X}_l^{\mathrm{t}} \geq 0$ is a relaxation component that fulfills the GP model, $\mathrm{X}_l^{\mathrm{t}} \in$ R, and l= 1 for R_l^0. As revealed in equations (10), the performance of the securities is only constrained which allows the portfolio to invest in bond when the investment in securities is not favorable. The second portfolio objective is to minimize the portfolio risk measured with beta. The value of optimal beta β^* is calculated by using first stage variables as the accompanying subproblem:

$$min\, \mu \sum_{s=1}^{n} \beta_s\, g_s^0 + (1-\mu) \left| \sum_{s=1}^{n} \emptyset_s^0 x_s - B_\beta \right| \tag{11}$$

Subject to

$$\sum_{s=1}^{n} \mathrm{g}_s^0 \leq \mathrm{G}^0 \tag{12}$$

$$x_s \leq C g_s^0;\ \forall\, s \in \Upsilon \tag{13}$$

$$x_s \geq 0\,, \qquad x_s \in \mathbb{I};\ \forall\, s \in \Upsilon \tag{14}$$

$$g_s^0 \in \mathbb{B};\ \forall\, s \in \Upsilon \tag{15}$$

Where 0< μ <1 and B_β the initial portion of portfolio invested in security. Accordingly, $\beta^* = \sum_{s=1}^{n} \beta_s\ (g_s^0)^*$, where $(g_s^0)^*$ is the optimal value resulted from solving the model (11) – (15). The equation (12) exhibited in the subproblem model to bound the quantity of security names in the portfolio as cardinality constraint. Since the calculation of $\beta^*$are in view of recorded price movement it gives the best perceived risk in the first time period t=0. The optimal value of risk β_{sl}^* of the security s at time t>0 is computed utilizing SP to facilitate future uncertainty of the market Then, the accompanying constraint is added to the model

$$\sum_{i=1}^{n} \beta_i\, g_i^0 \leq \beta^* + \delta^0 \tag{16}$$

For time t > 0 uncertainty must add to the optimal risk value by including scenarios. Thus, for time t > 0, the optimal security risk becomes β_l^* and when associated with single stock it becomes β_{il}^* as shown:

$$\sum_{i=1}^{n} \beta_{il}^t\, g_{il}^t \leq \beta_l^* + \delta_l^t;\ t = 0, \dots, \mathrm{T} \quad, l \tag{17}$$

Penalty variables δ^0, δ_l^t are minimized in the objective function. The third element, the liquidity exists in all financial investment, where common stocks are the most liquid. Liquidity cost is computed by the distinction between the purchasing cost paid by a rush purchaser and cost got by an exigent seller. Ranges in bid-ask spread determine the liquidity cost since brokerage firm commissions do not vary with the period of time taken to complete an exchange (Parra et al., 2001).

$$\text{Percent spread} = \frac{Ask - Bid}{Ask} * 100\% \tag{18}$$

As the financial specialist prefers to allocate his fund in highly liquid instruments, the liquidity solves for the optimal value Λ_l^* under each scenario, as same as the equations (11) - (15). In this way, the inequation (19) will be incorporated as a constraint in the main model of the problem,

$$\sum_{i=1}^{h} \Lambda_{(i,t,l)}\, g_{il}^{t} \geq \Lambda_{l}^{*} - \Lambda^{t} \qquad , \mathrm{t} = 0, \ldots, \mathrm{T} \quad , l = 1, \ldots, \mathrm{L} \tag{19}$$

where ¸ $\Lambda^t \geq 0$ is a penalty variable that is minimised in the objective function and accompanied by a penalty parameter. Finally, information will be considered as a last element in the portfolio selection model. Depending on the estimation conducted from historical data, the drift can be predicted using Brownian motion as mentioned earlier according to Valian (2009) and the work of Osborne (1962, 1972). Drift can be predicted using the equation in Figure 4.2. It is used in making decisions to be ensured that the decision is accurate by maximising the quality of available information. For this purpose, symbol (d or μ^t_i) represents the drift, Π_i represents the quality of information about security i at time t=0, and Π^t_{il} represents the quality of information about security i at time t ≥ 1, under scenario *l*. w_d, w_σ are the weight of drift and volatility respectively. The object is to maximise the value of Π_i and Π^t_{il}, the following two equations used to fulfil this object.

$$\Pi_i = (w_d\ 1/d_i + w_\sigma\ 1/\sigma_i),\ t=0 \tag{20}$$

$$\Pi^t_{il} = (w_d\ 1/d_{il}^{\ t} + w_\sigma\ 1/\sigma_{il}^{\ t}),\ t \geq 1 \tag{21}$$

As the investor desires to maximise the value of Π_i, the information will be solved to the optimal value Π^* for the two stages, and includes it as a constraint in the main model, where ς^0 is a penalty variable that is minimised in the objective function and accompanied by a penalty parameter, as follows:

$$\sum_{i=1}^{n} \Vert_{(i,0)}\, g_i \geq \Vert^* - \varsigma^0 \tag{22}$$

$$\sum_{i=1}^{n} \Vert_{(i,t)}\, g_{il}^t \geq \Vert^* - \varsigma_{il}^t \tag{23}$$

The ESGMIP model will be

$$\begin{aligned} Min\ \ -\mu_1 &\left(\sum_{l=1}^{L}\sum_{i=1}^{n} \text{Ø}_{il}^{1}\ x_i + \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{i=1}^{n} p_l\ \text{Ø}_{il}^{t+1}\ \mathrm{y}_{il}^{t} + \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{j=1}^{h} p_l\, U_{jl}^{t} z_{jl}^{t-h_j^*} \right) \\ &+ \mu_2\left(\mid \varsigma^0 \mid + \sum_{t=1}^{T}\sum_{l=1}^{L} p_l \ \mid \varsigma_l^t \mid \right) + \mu_3 \left(\sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{i=1}^{n} p_l\, \overline{w}_{il}^{t} \right) + \mu_4 \left(\sum_{s=1}^{S} \left|\xi_s^0\right| + \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{s=1}^{S} p_l \left|\xi_{sl}^t\right| \right) \\ &+ \mu_5\left(\delta^0 + \sum_{t=1}^{T}\sum_{l=1}^{L} p_l\, \delta_l^t \right) + \mu_6\left(\lambda^0 + \sum_{t=1}^{T}\sum_{l=1}^{L} p_l\, \lambda_l^t \right) + \mu_7\left(\chi^0 + \sum_{t=1}^{T}\sum_{l=1}^{L} \chi_l^t \right) \end{aligned} \tag{24}$$

Subject To

$$\hat{B} = \sum_{i=1}^{n} \text{Ø}_i^0 x_i + \sum_{j=1}^{h} \varphi_{jl}^0 z_j^0 \tag{25}$$

$$\hat{B}_l^1 = \sum_{i=1}^{n} \text{Ø}_{il}^1 x_i - \sum_{i=1}^{n} \text{Ø}_{il}^1 y_{il}^1 + \sum_{j=1}^{h} U_{jl}^1 Z_j^{1-h_j} - \sum_{j=1}^{h} \varphi_{jl}^1 Z_j^1 - \tau_l^1 \overline{w}_{il}^1\ , \quad l \in \Omega \tag{26}$$

$$\sum_{i=1}^{n} \text{Ø}_{il}^1 x_i \leq R^0 + \chi^0 \tag{27}$$

$$\sum_{i=1}^{n} \text{Ø}_{il}^{t} y_{il}^{t} \leq R_{l}^{t} + \chi_{l}^{t}, \quad \forall\, l \in \Omega, \qquad \mathrm{t} \in \bar{T} \tag{28}$$

$$\sum_{i=1}^{n} \mathrm{Q}_{(i,s)}\, \text{Ø}_{i}^{0} x_{i} = f_{s}^{0} \sum_{i=1}^{n} \text{Ø}_{i}^{0} x_{i} + \xi_{s}^{0}, \quad s \in S \tag{29}$$

$$\sum_{i=1}^{n} \mathrm{Q}_{(i,s)} \text{Ø}_{il}^{t}\, y_{il}^{t} = f_{s}^{t} \sum_{i=1}^{n} \text{Ø}_{il}^{t}\, y_{il}^{t} + \xi_{sl}^{t} \quad , l \in \Omega, \mathrm{s} \in \mathrm{S}, t \in \mathrm{T}, l \in \Omega, \mathrm{t} \in \mathrm{T} \tag{30}$$

Bound the upper bound # of Assets when t=0, t > 0

$$\sum_{i=1}^{n} g_{i}^{0} \leq G^{0} \tag{31}$$

$$\sum_{i=1}^{n} g_{il}^{t} \leq G^{t}, \quad \forall l \in \Omega, \mathrm{t} \in \bar{\mathrm{T}} \tag{32}$$

Bound # of bonds when t=0, t > 0

$$\sum_{j=1}^{h} \tilde{g}_{i}^{0} \leq \tilde{G}^{0} \tag{33}$$

$$\sum_{j=1}^{h} \tilde{g}_{il}^{t} \leq \tilde{G}^{t}, \quad \forall l \in \Omega, \mathrm{t} \in \bar{\mathrm{T}} \tag{34}$$

$$\sum_{i=1}^{n} \beta_{i}\, g^{0} \leq \beta^{*} + \delta^{0} \tag{35}$$

$$\sum_{i=1}^{n} \beta_{il}^{t}\, g_{il}^{t} \leq \beta_{l}^{*} + \delta_{\mathrm{l}}^{\mathrm{t}}, \quad l \in \Omega, t \in \bar{\mathrm{T}} \tag{36}$$

$$\sum_{i=1}^{n} \Lambda_{(i,0)}\, g_{i} \geq \Lambda^{*} - \lambda^{0} \tag{37}$$

$$\sum_{i=1}^{n} \Lambda_{(i,t,l)}\, g_{il}^{t} \geq \Lambda_{l}^{*} - \lambda_{l}^{t}, \quad \forall\, l \in \Omega, t \in \bar{\mathrm{T}} \tag{38}$$

$$\sum_{i=1}^{n} \Vert_{(i,0)}\, g_{i} \geq \Vert^{*} - \varsigma^{0} \tag{39}$$

$$\sum_{i=1}^{n} \Vert_{(i,t)}\, g_{il}^{t} \geq \Vert^{*} - \varsigma_{il}^{t} \tag{40}$$

$$\bar{w}_{il}^{1} = |y_{il}^{t} - x_i| \,, \ \forall\, i \in \Upsilon, l \in \Omega \tag{41}$$

$$\bar{w}_{il}^{t} = |y_{il}^{t} - y_{il}^{t-1}|;\ \forall\, i \in \Upsilon, l \in \Omega, \mathrm{t} \in \widetilde{\mathrm{T}} \tag{42}$$

$$x_i \leq C\mathrm{g}_{\mathrm{i}}^{0}\,;\forall\ i \in \Upsilon \tag{43}$$

$$y_{il}^{t} \leq C\mathrm{g}_{il}^{\mathrm{t}};\forall\, i \in \Upsilon, l \in \Omega, t \in \overline{\mathrm{T}} \tag{44}$$

$$Z_j^{0} \leq C g_j^{o}\,;\ j \in \Xi \tag{45}$$

$$Z_{jl}^{t} \leq C\tilde{g}_{jl}^{t}\,;\ \forall\, j \in \Xi, l \in \Omega, t \in \overline{\mathrm{T}} \tag{46}$$

$$x_i \leq d_i\ ,\quad \forall\ i \in \Upsilon \tag{47}$$

$$\mathrm{y}^{\mathrm{t}}_{\mathrm{il}} \leq \mathrm{d}_i\ \forall i \in \Upsilon,\ l \in \Omega,\ t \in \mathrm{T} \tag{48}$$

$$Z_j^{0} \leq \tilde{d}_j\,;\ \forall\, j \in \Xi \tag{49}$$

$$Z_{jl}^{t} \leq \tilde{d}_j\,;\ \forall\ j \in \Xi, l \in \Omega, t \in \tag{50}$$

Non-negative Constraints

$$x_i \geq 0; x_i \in \mathrm{R},\ i \in ? \tag{51}$$

$$y_{il}^{t} \geq 0,\ \ y_{il}^{t} \in \mathrm{R}, \forall\, i \in \Upsilon, l \in \Omega, t \in \overline{\mathrm{T}} \tag{52}$$

$$Z_j^{0} \geq 0\,,\quad Z_j^{0} \in \mathrm{R}\quad \forall\, j \in \Xi \tag{53}$$

$$Z_{jl}^{t} \geq 0\,,\quad Z_{jl}^{t} \in \mathrm{R}\quad \forall\, j \in \Xi,\ l \in \Omega, t \in \overline{\mathrm{T}} \tag{54}$$

$$\bar{w}_{il}^{t} \geq 0,\quad \bar{w}_{il}^{t} \in \mathrm{R}\quad \forall\, i \in \Upsilon, l \in \Omega, t \in \overline{\mathrm{T}} \tag{55}$$

$$\delta^{0} \geq 0,\qquad \delta^{0} \in \mathrm{R} \tag{56}$$

$$\delta_l^{t} \geq 0\,,\qquad \delta_l^{t} \in \mathrm{R}\ \ \forall\ l \in \Omega,\qquad t \in \overline{\mathrm{T}} \tag{57}$$

$$\lambda^{0} \geq 0,\ \lambda^{0} \in \mathrm{R} \tag{58}$$

$$\lambda_l^{t} \geq 0,\qquad \lambda_l^{t} \in \mathrm{R}\qquad \forall\ l \in \Omega, t \in \overline{\mathrm{T}} \tag{59}$$

$$\Pi_i \geq 0,\qquad \Pi_i \in \mathrm{R}\qquad \forall\, i \in ? \tag{60}$$

$$\Pi_i^{t} \geq 0,\qquad \Pi_i^{t} \in \mathrm{R}\qquad \forall\, i \in ? \tag{61}$$

$$g_i^{0} \in B,\qquad \tilde{g}_j^{0} \in B\qquad \forall\, i \in \Upsilon,\qquad j \in \Xi \tag{62}$$

$$g_{il}^{t} \in B,\qquad \tilde{g}_{jl}^{t} \in B\qquad \forall\, i \in \Upsilon,\qquad j \in \Xi\,,\ t \in \overline{\mathrm{T}} \tag{63}$$

$$\xi_s^{0} \in R,\qquad \xi_{sl}^{t} \in \mathrm{R}\qquad \mathrm{s} \in \widehat{\mathrm{S}},\ l \in \Omega,\qquad t \in \overline{\mathrm{T}} \tag{64}$$

(Stoyan, 2011)

Equations$(25) - (26)$ revealed the initial investment amount constraints for first and second period. In $(27) - (28)$ the passive strategy used to constraint the performance of portfolio will not exceed market portfolio, when t=0, t > 1. At the same time, the performance of stock is only constrained which permitted the portfolio to invest in bond when the investment in stock is unfavorable. The equation (29) and (30) set up the first and second stage sector constraint, respectively. Equations (31) and (32) set for upper bound on the stock to hold, G^t in first and second stage. $\tilde{G}^{t}$ is the upper bound on the bond to hold as shown in (33) and (34). In equation (43)-(46), C is a large constant to limit x_i, y_i, Z_j^0, Z_{jl}^t as binary decision variables, In the next sections the model implementation is discussed.

3. Model Implementation

A specific model is designed for solving the ESGMIP model presented in (24)-(64). A decomposition algorithm used to facilitate finding the solution. The basis on which the algorithm operates depends on adding penalty variables, especially for the most difficult constraint, which is the name to hold constraint and relaxed it in the objective function by adding penalty parameter. The algorithm decomposes the problem into stock and bond subproblems, and then goes through sector decomposition. The subproblems are solved individually and the resulted values used in the master problem (24) to (64) to check for optimality, (see Figure 1). The model either reaches the optimality or needs adjusting the initialization.

The robustness of the designed algorithm refers to the model decomposition into subproblems. The relaxation added to the difficult constraint - cardinality constraint, G^t - is being the other strength of the algorithm, which is one of the concentrates in this study. The cash balancing constraint Bt, the parameter that defines the upper bound on the number of securities to hold in the portfolio Gt, and the portfolio benchmark Rt, were distributed between the market sectors then collected in the master problem by setting

$$\mathrm{B}^{\mathrm{t}} = B_1^t + B_2^t + B_3^t \tag{65}$$

$$\mathrm{G}^{\mathrm{t}} = G_1^t + G_2^t + G_3^t \tag{66}$$

$$\mathrm{R}^{\mathrm{t}} = R_1^t + R_2^t + R_3^t \tag{67}$$

The solutions of the subproblems are directed to the master problem as an initial solution. The master problem in its turn assesses the optimality of the solution by checking if the portfolio goals fall within the benchmark criteria (66) to (67) and hold the same constraints. Then, either acknowledge (accept) the optimal solution or need adjustments by expanding the parameters related with the variables that fall beneath the criteria and resolves. Due to generating the initial solution by decomposition, the ESGMIP solves quickly.

Next, this study considered the systematic risk associated with individual security in the sample portfolio to achieve the objective function of minimizing the portfolio risk. The beta-coefficient, β is a measure of the systematic risk according to the financial literature (Al-Tamimi, 2010). It is the volatility of the individual security return over the time period compared to the movements of the market benchmark (Chan, 1992). Considering systematic risk, the (15) and (16) constraint is added. The optimal β^* value, which is also solved in a separate subproblem described in equations (11)-(15).

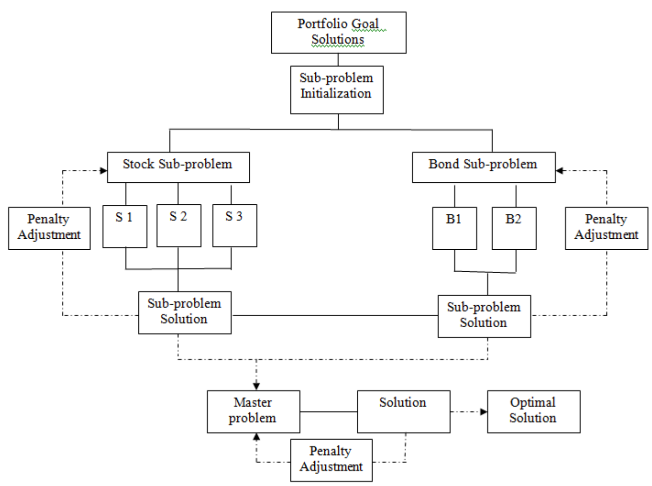

Figure 1. Chart Flow of ESGMIP decomposition

4. The Results of ESGMIP

This paper presents the results of solving the two-stage mathematical model which are analysed using the appropriate estimation procedures. All listed and continuously traded companies in the ASE and its distribution across market sectors are used in the stochastic model to achieve the objectives of this study. The stochastic model includes 100 securities traded in the ASE from the beginning of January 2010 to the end of December 2014. Also, it includes all issued bonds over the same time period, which amount to be two Zero-Coupon rate bonds. The daily characteristics of all listed and continuously traded securities are compared with the market portfolio ASE100 for comparison and clarification purposes. The ASE consists of three general sectors which are financial, industrial and service. Considering all these issues, the ESGMIP contained over 518 decision variables and 791 constraints. The SGMIP problem is solved using the previously mentioned algorithm with IBM ILOG CPLEX software version 12.7, on Intel ® core, 2.53 GHz i3 CPU. The decomposition of the algorithm improved the memory allocation and CPU timeThe results of the ESGMIP are presented after implementing the decomposition based on the market sectors to pure stocks portfolio as well as stock and bond portfolio.

4.1 Results of Pure Stocks ESGMIP Portfolio

The results of the ESGMIP portfolio using the CPLEX Ver. 12.7 are compared to the ESGMIP algorithm designed in Section 3. The comparison is based on the solution quality and the solution CPU time of the two portfolios. When running the CPLEX with either pure stock portfolio or stock and bond portfolio, the CPU time exceeded 21:22:45:15 hours and could not solve the model. Therefore, the designed algorithm is implemented and applied to the pure stocks ESGMIP portfolio. The ESGMIP portfolio outperforms the index portfolio in total return. This indicates that managing the drifts of each stock in the portfolio helps the algorithm model select the optimal portfolio according to the portfolio manager’s preferences.

4.1.1 Computational Results

After implementing the algorithm, the second stage of the equations is applied to the pure stocks portfolio. The ESGMIP algorithm decomposed portfolio reached the optimal solution and achieved the objective function within 00:00:16:05 seconds. The CPLEX runs for 0:7, 4:18 and 11:80 seconds for the financial, services and industrials sectors respectively. The best time achieved by bank sector is 00:00:00:07 seconds. The worst time was 00:00:11:80 seconds when running the industrial sector algorithm. At the first stage, the ESGMIP portfolio outperforms the index portfolio in total return, where ESGMIP return is 72.7% comparing with 17% of Index return, as shown in Table 1. For the second stage which comprises of three scenarios best L_b, stable L_I and worst L_w. The return distributed between scenarios as follows; 31.8% for the best, 30.6% for the stable, and 29.2% for the worst. The two-stage ESGMIP pure stock portfolio outperforms the Index portfolio. The ESGMIP dynamic portfolio did better than the previous study of Alhalaseh (2018) which used SGMIP without considering the information variable by 136.8% in total return. This indicates that managing the drifts of each stock in the portfolio helps the algorithm model select the optimal portfolio according to the portfolio manager’s preferences.

4.1.2 Financial Results

The financial results of the ESGMIP pure stock dynamic portfolio were acquired by comparing the performance of the ESGMIP portfolio to the performance of the market index. In the first stage, the daily return of the ESGMIP pure stock portfolio is distributed between the market sectors i.e., financial, services and industrials. The performance of the portfolio for each sector exhibited 14.5%, 66.8% and -7.9 %, respectively. The return of ESGMIP pure stock portfolio exceeds the market portfolio by 327.6% as shown in Table 1. Figure 2 revealed the superiority of the extended portfolio performance than the Index portfolio in daily return.

Table 1. Performance of ESGMIP pure stock portfolio/ stage one

	Financial Sector	**Service Sector**	**Industrial Sector**	**Portfolio**
ESGMIP pure Portfolio	0.145	0.66	-0.079	0.727
Index Portfolio	0.019	0.033	0.008	0.17

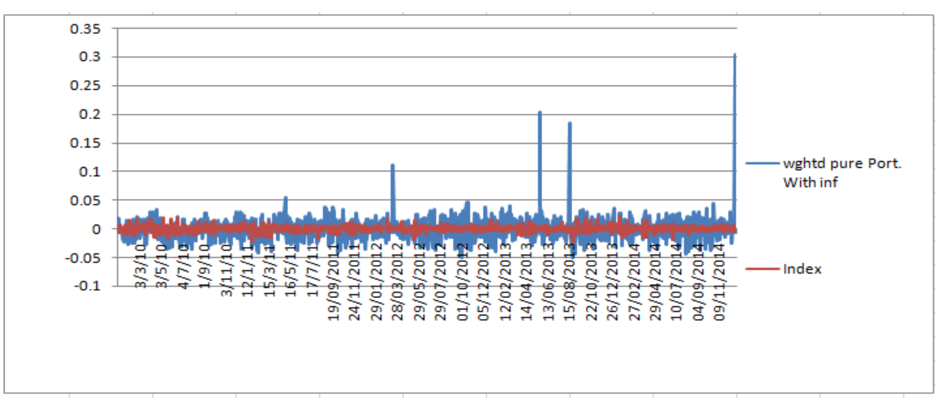

Figure 2. The daily return of the portfolio after information and the Index

In the second stage, the three scenarios are best, stable, and worst. In the worst scenario the ESGMIP portfolio return is aggressively fluctuates compare to the index. The ESGMIP portfolio achieved 29.2% total return which is greater than the total return of the index portfolio. This result may refer to the beta value of the ESGMIP portfolio. The high value of the beta seems to be the reason behind the high return, in line with the direct relationship between risk and return. Comparing ESGMIP portfolio performance with SGMIP portfolio performance the difference was negligible. The value of the upper bound of the name-to-hold, G was 24. The model manages one exchange for each year to rebalance the portfolio weights, as mentioned before.

Interestingly, the results from the portfolio algorithm designed in the first stage found almost no difference in the performance between the SGMIP (general portfolio and single scenario) and index portfolio, as discussed above. An improvement in the portfolio return is achieved after adding the information (drift) objective. The portfolio gain in the second stage appeared to be connected to the algorithm design (speed up) and uncertainty condition. Figure 2 reveals the results of the second stage between the worst scenario of the SGMIP portfolio performance and the index portfolio. In the second stage, the worst scenario takes place in July 2010 where the index portfolio overtakes the SGMIP portfolio by 0.03, as illustrated in Figure 3. These results can be enhanced for some months if the best scenario is taken, which is presented in Figure 4.

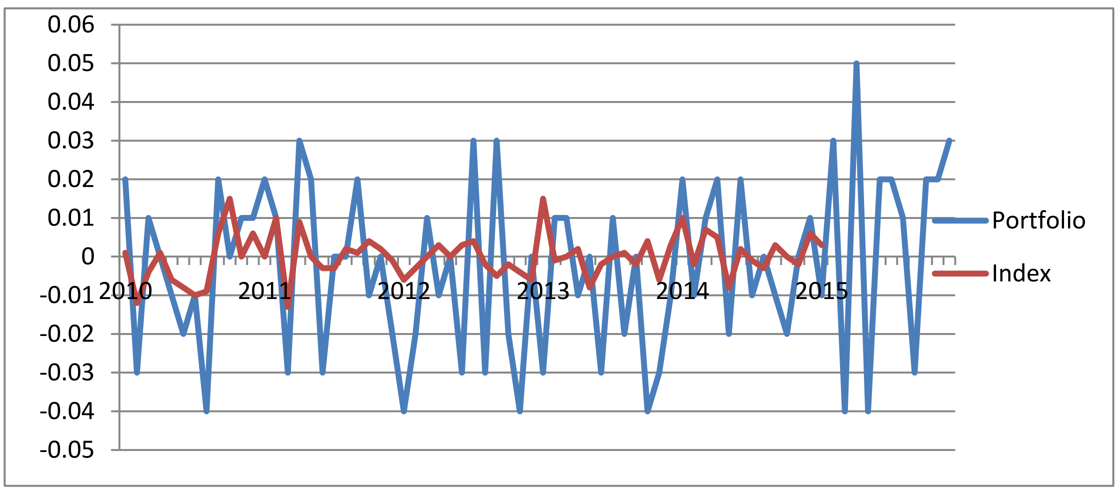

Figure 3. Worst case SGMIP portfolio compare with index- second stage

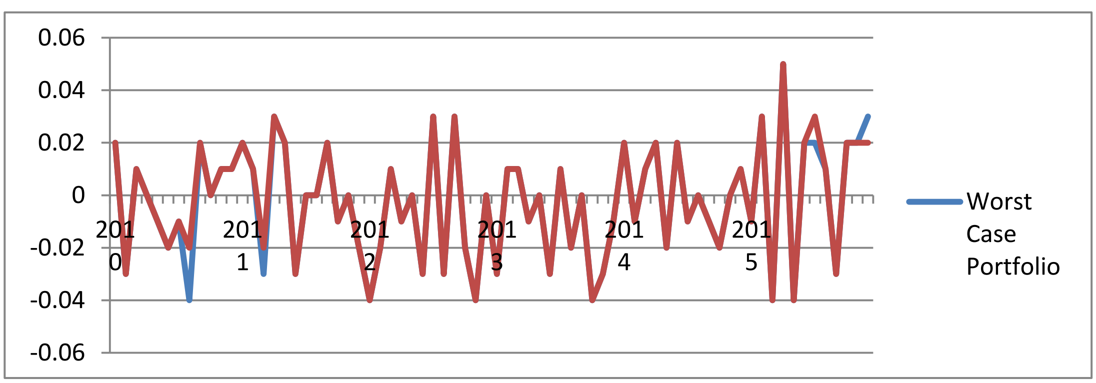

Figure 4. Best and worst case comparison- second stage

4.2 Results of Stocks and Bonds ESGMIP Portfolio

This subsection includes the bonds sample to the dynamic pure stocks portfolio and applies the decomposition algorithm. The performance of the resulted portfolio is displayed in the first and second stages as a dynamic portfolio.

4.2.1 Computational Results

For SGMIP stock and bond portfolio, the first stage contains stocks only. The performance of the sample stock and bond portfolio that includes all the objectives and decomposed by sectors equals 31.8%. The performance of the sample ESGMIP stock and bond portfolio is greater than the performance of the Index ASE portfolio. This portfolio consists of 15 name-to-hold stocks and does not invest in bonds. It consists of six stocks from the financial sector, five stocks from the service sector, and four stocks from the industrial sector.

The second stage was added to ESGMIP stock and bond portfolio to obtain the stock and bond dynamic portfolio. The CPLEX runs to solve ESGMIP stock and bond portfolio. It lasts for 00:97 seconds. These CPU times are less than the best case of Stoyan (2009). The difference between best case CPU time of Stoyan (2009) and CPU time of this model is 2:09:77:02 seconds in the first portfolio and 2:09:92:10 seconds for the second portfolio.

As mentioned before, the three scenarios are l_b, l_I, and l_w. The portfolios of the best l_b and stable l_I scenario invest in stocks and bonds, while the worst l_w invests in bonds only. The best scenario invests in seven name-to-hold securities and two different bonds. The stable scenario invests in eight name-to-hold securities and two different bonds.

4.2.2 Financial Results

The financial results for this study are obtained by comparing the performance of the ESGMIP algorithm pure stock portfolio and stock and bond portfolio with the performance of the index portfolio. After calculating the return of the two dynamic portfolios, the result shows that the performance of algorithm ESGMIP portfolio of pure stocks 72.7% outperforms the ESGMIP portfolio of stocks and bonds by 1.3% times and the index in the first stage as it contains pure stocks. Comparing these results of the extended portfolios ESGMIP with Alhalaseh (2018) SGMIP, the results indicated that the performance of the first stage ESGMIP portfolio outperform the performance of the ESGMIP portfolio by 3.6%.

When investigating the second stage, the l_b scenario achieved 10.6% total return from investing in security and 5.091% and 5.174% from bonds (see Table 2). The l_I portfolio achieved -12.2% total return from investing in security and 5.091% and 5.174% from the bonds. The algorithm ESGMIP portfolio of pure stocks outperforms the ESGMIP portfolio of stocks and bonds after decomposition in the best, stable and worst scenarios. The ESGMIP portfolio in the best scenario outperforms SGMIP portfolio in Alhalaseh (2018) by 333.04%. In the stable and worst scenarios the difference was negligible. The ESGMIP pure stock and stock-bond portfolio are greater than the ASE100 portfolio in total return.

Table 2. Performance of the ESGMIP portfolios

Portfolio	Stage1	Stage 2		
Type		L_B	L_I	L_W
Pure Stock Port.	72.7%	31.8%	30.6%	29.2%
Stock-Bond Port.	31.8%	10.6%	(12.2)%	---
Bonds average return		5.13%	5.13%	5.13%

The SGMIP stock and bond portfolio performance of best and stable scenarios is 15.7% and -7.07% (bond return 5.13% added). This is a logical consequence of the decline in return when the investment conditions become uncertain. The best scenario remained higher than the index portfolio (see Table 5.8). A fascinating result is in the worst scenario where the portfolio invested the entire allocated amount in bonds only. This fascinating result satisfied the objective of the SGMIP model in investing in mixed stocks and bonds portfolio. The objective is to allow the portfolio to abandon risky assets to safe investments in bond.

This research evaluates the total risk, σ of the extended SGMIP model portfolios. The float Index of ASE100 is used as a benchmark to fulfil the evaluation and prove the results of this study financially. The monthly total risk of ESGMIP pure stock portfolio return, ESGMIP stock and bond portfolio return and index portfolio return are 0.026, 0.0036 and 0.0013 respectively. Figure 5 portrays the monthly standard deviation of the portfolios which revealed that the market portfolio ASE100 has a lower total risk than the pure stock portfolio and the stock and bond portfolio.

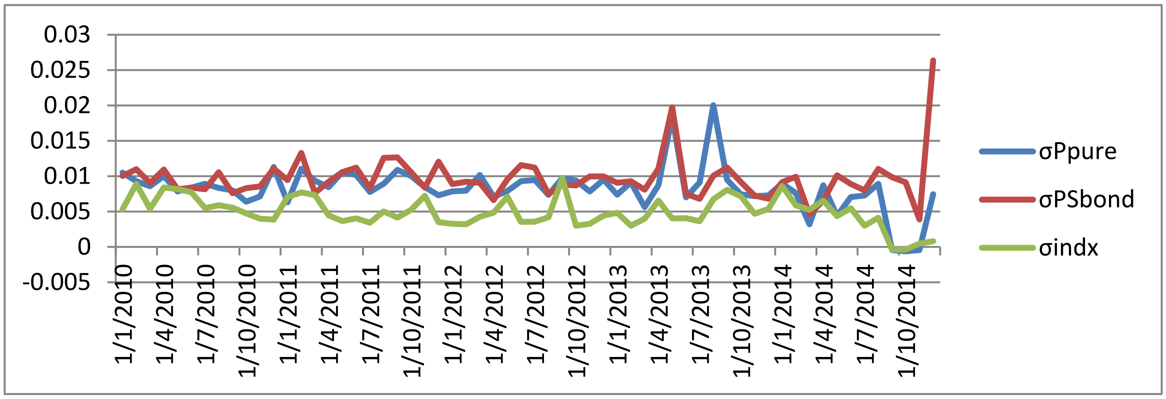

Figure 5. Monthly standard deviation of the portfolios

Furthermore, for the part of minimising the portfolio risk, the optimal value of beta from running the optimal beta β^* subproblem equations 11 to 15 on CPLEX using Optimisation Programming Language (OPL) is 1.007. As such, Table 3 reveals that beta of the algorithm pure portfolio in the first stage decomposed by sector equal to 0.66, 0.39, and 0.70 respectively. The average beta of the three sectors is 0.58 (minimise the beta). The beta of the ESGMIP stock and bond portfolio in the first stage is 0.68 (minimise the beta). In the second stage, the algorithm of the pure portfolio invested in banking sector reveals differences between the three scenarios.

Table 3. CPLEX output of weighted portfolio beta/ stage two

Portfolio	Sector	First Stage	Second Stage		
			L_B	L_I	L_W
Pure Stock	Financial	0.66	0.74	-0.17	-2.50
	Service	0.39	0.24	-0.80	-0.98
	Industrial	0.70	0.23	0.19	0.58
Stocks and Bonds		0.682	0.559	-1.38	---

Considering the goal of limiting number of stocks to hold (the cardinality constraint) of the pure portfolio, the beta of the banking sector is 0.74 in the best scenario. The beta takes the same direction of the market and minimise the beta value by 35%. In the worst scenario, the beta of the portfolio is against the market direction with the greatest value of beta -2.50. The beta of the service sector is far from the beta of the other two sectors over the three scenarios. The portfolio of the industrial sector was more defensive than the banking and service sectors. The beta of the industrial sector in the first stage is 0.70 less than the market's beta. In the second stage, the portfolio becomes more defensive with beta 0.58 in the worst scenario.

The SGMIP stock and bond portfolio attains a very interesting result where it invested with beta 0.560 in the best scenario, took more risk -1.38 in the normal scenario, and turned to invest completely in bonds when the investment environment became unfavourable. These results prove that the extended SGMIP model with new and larger practical managing constraints can select the dynamic portfolio practically and minimise the risk in the context of ASE. The features of the model can help the portfolio manager to select the optimal dynamic portfolio according to the investors' preferences.

Other two questions need to be answered to satisfy the research objectives. The first one is what is the liquidity value of the selected dynamic optimal portfolio? The answer is based on the results of the CPLEX solution of the ESGMIP model. The optimal value of liquidity Λ^* was calculated in subproblem equations as mentioned equations 11 to 15. OPL programming language was used to write the program. CPLEX was then ran for the subproblem. The resulted optimal value Λ^* was inserted in the ESGMIP main model displayed in equations 24 to 64 and minimises the liquidity penalty λ^0,λ_l^t in the objective function equation 24. The ESGMIP stock and bond dynamic portfolio liquidity is equal to the optimal 3.8572, where the penalty was equal to zero. In the higher time period, the liquidity penalty increased to 0.4915, 0.4106, and 0.3933. The results indicated that the liquidity was affected by the uncertainty condition in the higher time period. The portfolio become less liquid in time *t* since it invests in bonds. The trades in the bond market in the ASE are weak since the market opened for trade once monthly and the studied type of bond is of zero-coupon rate (Al-Tamimi, 2010).

The investor pays fees and commission or transaction costs when securities are bought or sold between periods. The ESGMIP model was designed to hold long-term portfolios aiming to reduce the transaction cost. Equation 41 is used to constrain the number of the transactions between time periods. To answer the second question "What is the transaction cost that the investor will pay to get the selected dynamic portfolio?" This study refers to the results in the ESGMIP model for each portfolio. In the current time t_0 there is no transaction cost. The cost will occur when rebalancing to a higher time period t_1. Because of uncertainty, the ESGMIP model established three scenarios. In the stock and bond portfolio, if the situation is favourable l_B the agent executed complete share selling of 13 names-to-hold stock, partial selling of two names-to-hold stocks and buying five new names-to-hold stock transactions. Thus, the resulted transaction cost is 791.94 JD. When there is stability in the environment $l_{I,}$ the portfolio agent will execute 13 stocks complete selling, one partial selling of name-to-hold stock, six buying new names to hold stock and hold one name-to-hold stock. This process costs the investor 611.27 JD. When the investment environment is unfavourable l_w, the investor withdraws from the market by selling the entire portfolio of securities and investing the available amount in bonds. The investor will pay 585.83 JD to execute15 selling of name-to-hold stocks, in addition to 22.5 JD as a bond transaction cost. In the pure stock portfolio, when the investor faced favourable environment l_B, he executes just two selling transactions. It will cost him/her 127.6 JD while hold the remaining 14 name-to-hold stocks. In l_I, three selling transaction costs 156.5 JD, and in l_w, also three selling transactions costs 191.3 JD, where the transaction Cost= Difference*0.0054). It is clear that the SGMIP model in its design succeeded in managing the transaction cost between time periods. This is discussed further in next section.

5. Portfolio Discussion

This study introduced a selection model for a complex portfolio and offered a solution method that solved the multi-objectives, multi- assets, and multi-stages problem under, the uncertain condition in the financial markets. Referring to the computational result sections, the stock-bond portfolio achieved incumbent optimal solution and the pure stock portfolio achieved optimal solution comparing with Stoyan (2009) and Stoyan and Kwon (2010, 2011). Also the ESGMIP algorithm model outperforms the SGMIP algorithm model of Stoyan and Kwon (2011) regarding CPU time to solve the portfolio problem by 4:95 seconds. This indicates that the ESGMIP algorithm efficiently solved the large-scale portfolio problem containing multi-objectives and captured financial uncertainty in the market.

Regarding the financial results, including different decision variables and investing in the stock and bond investment allowed the portfolio to follow a long-strategy that outperforms the market portfolio of ASE. The good results in the portfolio return in all scenarios indicate the robustness of using the time average value based on daily return in predicting the assets price in the stochastic portfolio rather than the expected return used by the traditional models such as MV. Also, the results of pure stock portfolio show a return drop in the worst scenario compared to the other two scenarios while the portfolio achieved a higher beta. It is a sign that the volatility in the ASE refers to the behaviour or temperament of its investors rather than the reason behind this behaviour. The fascinating result of stock and bond portfolio occurred when the portfolio turned to use the entire investment amount in bond in the worst scenario. This result refers to the ESGMIP model design and the uncertainty conditions. As Topaloglou et al. (2008) demonstrated, the SP framework grants a flexible and effective decision support tool for portfolio management. By comparing these results with Alhalaseh (2018), Stoyan and Kwon (2011), Stoyan (2009) and Konno and Kobayashi (1997), the contribution of adding additional managing characteristic (information) to the dynamic multi-objectives is attained.

6. Implications

The findings add to the knowledge an extending mathematical model ESGMIP and algorithm solution that effect the dynamic portfolio selection, providing empirical evidence in selecting and solving the portfolio problem as a real world problem. The materials presented in this research are considered a novelty development of knowledge in PSP as an important area in the financial engineering and investment. Tthe portfolio managers all over the world can use the goal sub-model to get the optimal values of their portfolio goals according to the context of their investment. They can use the method and the model to rebalance or evaluate their portfolio at least on a daily base to overcome the uncertainty accompanying the financial markets.

7. Conclusion

This research presents a mathematical approach to complex financial problems. Financially, the mathematical model contains various numbers of conflicting goals and manages many constraints. The contributions of this study manifested in extending financial model by adding a new characteristic (information) that does not investigate as portfolio managing characteristic before, examine pure stock portfolio and stock-bond portfolio, a method to capture the uncertainty in the financial markets, and designing a special mathematical algorithm to solve the extended complex financial using daily data. A fascinating result is obtained from the portfolio algorithm design. An improvement in the portfolio return is achieved after adding the information (drift) objective. The information factor as portfolio characteristic was capable of improving the portfolio selection and headed the benchmark return.

For further investigations, this study suggests enhancing the model. It can be adjusted to absorb other investment tools such as riskless or near riskless assets as an extension of the options for investors. Increasing the complexity of the model design by adding more constraints and scenarios will force the using of decomposition algorithm that may need to be tailored. One can extend the portfolio for multi-periods by rebalancing the portfolio monthly, quarterly, and semi-annually, or for periods that exceed one year. Also, one may examine the designed model's outcomes statistically. Moreover, researchers could apply the model to other financial markets. The limitation that faced this research represented in unavailability of some data such as liquidity for each stock. where the study found it difficult to obtain the intraday data of the trades because such trades not registered by the market and its institution. In conclusion, these results prove that the extended model of the large-scale portfolio selection problem solved optimally and efficiently within the short time.

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