

Is S&P100 Index a Mean-Variance Efficient Portfolio?

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Abstract

This paper investigates whether Standards and Poors' 100 stock index (henceforth, S&P100) is a mean-variance efficient portfolio. In other words, the paper examines if there is any other portfolio allocation rule that provides better risk-return scenario than one that is achieved under value-weighted strategy used in S&P100 index. Our results show that the realized risk of the S&P100 index is not the lowest for the return at any other portfolio combinations using all stocks in index. There are other portfolio combinations that provide lower risk for the same realized rate of return. Similarly, the realized return of S&P100 index is not the highest for the standard deviations at any other portfolio combinations. Based on these results, we conclude that the S&P100 index may not be considered to be a mean-variance efficient portfolio.

Keywords: S&P100 index, mean-variance efficiency, portfolio

1. Introduction

To select a portfolio, Markowitz (1952) and Tobin (1958) proposed that risk-averse investors must follow the mean-variance principle to maximize their utility. Under this rule, investors choose any alternative portfolio that includes a set of securities available in the market that can be combined in a linear combination in infinite possible ways. These chosen portfolios will include the portfolios that are efficient in the context of mean and variance. In other words, investors will discard those portfolio combinations that have lower mean and higher variance compared to any other given member of the set of portfolios. Under the mean-variance efficiency rule, a rational investor will select a portfolio that provides highest return for a given amount of risk or lowest risk for a given amount of return. This selection rule guarantees that investors are always better-off in the portfolio decision-making process. Fama (1965) defined the efficient portfolio as a portfolio that has the minimum variance for any given rate of return or that has maximum return for any given variance. Efficiency of a portfolio can be measured by evaluating its risk and return relationship over a period of time.

In this paper, we use the Standard and Poor's 100 stock index (henceforth, S&P100) to estimate the risk and return relationship by using its value-weighted portfolio allocation method. S&P100 index contains the top 100 companies from the US stock markets in terms of their market value. It is well-known that the index composition is revised from time to time to accurately represent the value of the overall market.

Our purpose is to empirically investigate if the S&P100 index, is in fact, a mean-variance efficient portfolio. According to the argument presented by Markowitz (1952), Tobin (1958) and Fama (1965), if the S&P100 index is a mean-variance efficient portfolio, its value-weighted rate of return should be the highest at the realized risk level compared to portfolios constructed under any other weighting scheme or its standard deviation should be lowest under value-weighted method at the realized return compared to portfolios under any other weighting scheme. That means, there should not be any other combination of those stocks in the S&P100 that should provide higher return than the realized return of the index at the same risk level or there should not be any other portfolios that should offer lower standard deviation at the same return realized from S&P100 index. The purpose of this paper is to examine whether there is any other portfolio allocation rule that provides better risk-return scenario than one that is achieved under value-weighted strategy. By using the daily return for all stocks in the index, we estimate the annualized return and risk for the portfolio that includes all stocks in the S&P100 index.

The empirical literature surprisingly does not directly address the performance of this very popular stock index. However, there have been a large body of research on the conditional mean-variance efficiency of U.S. and other

stock markets. For example, Engel, et. al. (1995) study the mean-variance efficiency of U.S. stock market by applying a method that allows conditional expected returns to vary in a relatively unrestricted way. They reject the MVE model against the Tobin model with the interpretation that the asset returns can be forecast *ex ante* in part by the asset shares and the relationship is not determined by the mean-variance optimization. Engel and Rodrigues (1993) also test the mean-variance efficiency of international equity markets. By using two different sets of tests (Wald test and asset demand equation constrained by MVE), they argue against the mean-variance efficiency of the world equity markets. Castellano and Cerqueti (2014) study the mean-variance portfolio selection process for infrequently traded stocks. They conclude that the long-term investment strategy outperforms the short-term strategy for negatively correlated stocks and in all other cases, short-term strategy performs better. They also argue that the results of the portfolio selection are in line with the diversification principle of Markowitz even when the thinly traded stocks are considered.

2. The Theoretical Framework

Mean-variance efficient portfolio theory is one of the basic fundamental concepts of modern financial theory. A rational investor always seeks to find a portfolio that has a minimum risk for a given rate of return or a portfolio that has the highest return for a given level of risk. This may indicate that some investors may want to minimize risk while others may decide to maximize return. This decision may, of course, depend on the investors' overall strategy depending on his/her risk tolerance and investment objectives. In order to achieve this efficiency, investors make careful decisions of their asset allocation.

Over the last few decades, finance theories have evolved significantly and a strong body of knowledge is available for practical application. Finance literature has been devoted to establish a plausible theory on the minimum-variance portfolio that is also known as efficient frontier.

An efficient frontier is determined by minimizing the risk at any level of expected returns. Thus to achieve efficient frontier at each level of return, the risk for each of those return levels should be minimized subject to the condition that sum of proportions invested in all assets be equal to one, and all securities have some positive investments. (Note 1) The second condition prohibits short sales.

The derivation of efficient portfolio involves minimizing the variance (σ^2) for a given level of expected return, $E(r)$. The problem can be expressed as follows:

$$\text{Minimize} \quad \sum_{i=1}^N W_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1, i \neq j}^N W_i W_j \sigma_{ij} \quad (1)$$

$$\text{Subject to} \quad (1) \quad \sum_{i=1}^N W_i = 1 \quad (2)$$

$$(2) \quad W_i > 0 \quad \text{and} \quad W_i < 1, \quad i = 1, \dots, N \quad (3)$$

Where, W_i is the proportion invested in each stock, σ_i is the standard deviation of return for stock i , σ_{ij} is the covariance between stocks i and j .

If a portfolio is efficient in terms of its mean-variance structure, then it should have the lowest variance (or standard deviation) for every possible rate of return for different combinations of investment proportions.

Our objective is to apply the theory of efficient portfolio to test whether the S&P100 index is a mean-variance efficient portfolio. For this purpose, we compare theoretical mean-variance scenario for the complete S&P100 index under different alternative asset allocations with the actual return of same index during 2012. We argue that there may be a portfolio combination that generates higher return and/or lower risk compared to the value-weight that is used to calculate the S&P100 index return and variance.

3. Data and Methodology

Daily stock price data for all stocks in the S&P100 index in 2012 were downloaded from Yahoo! Finance website for calendar year of January through December. As it is well-known that components of the S&P100 index change from time to time during the year, we decided to use the stocks that were in the index at the end of the year. (Note 2) From these price data, we first compute the daily returns for all stocks. Using these return data, we compute the geometric mean and variance of returns of each stock. We also develop the variance-covariance matrix for all stocks. Then we randomly generate portfolio weights for all 100 stocks by using the RAND (.) function in excel and repeat this process 100 times to create 100 portfolios. Now we have 100x1 return matrix, 100x100 variance-covariance matrix

and 100x100 weight matrix to compute the expected index return and variance by using each round of portfolio weights. The estimation of portfolio expected return and portfolio variance are shown below:

The specification of portfolio expected return takes the following form:

$$E(R_p) = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_{100} \end{bmatrix} X [W_1, W_2 \dots \dots \dots W_{100}] \tag{4}$$

where, $E(R_p)$ is the expected portfolio return, R_1, R_2, \dots, R_{100} are the computed geometric mean return of each of the individual stocks in S&P100 index, and W_1, W_2, \dots, W_{100} are the randomly generated weights assigned for each stock.

Variance of the portfolio takes the following format:

$$\sigma_p^2 = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_{100} \end{bmatrix} X \begin{bmatrix} \sigma_{1,1} & \dots & \sigma_{1,100} \\ \vdots & \ddots & \vdots \\ \sigma_{100,1} & \dots & \sigma_{100,100} \end{bmatrix} \tag{5}$$

where, σ_p^2 is the portfolio variance and $\sigma_{1,1}$ through $\sigma_{100,100}$ is the variance-covariance matrix for 100 stocks in the S&P100 index.

$E(R_p)$ and σ_p^2 have been calculated for 100 combinations of portfolio weights that were randomly generated with excel RAND function. This process generates one hundred combinations of $E(R_p)$ and σ_p^2 . These portfolio returns are sorted from highest to lowest with corresponding variances and similarly the portfolio variances are sorted from the lowest to the highest with corresponding returns. Then we identify the minimum variance and the corresponding return from the list of the sorted σ_p^2 . Then we separate the returns that are higher than the return corresponding to the minimum variance. The remaining combinations are discarded since those either have lower return and/or higher variance compared to the minimum variance portfolio. Our next task is to isolate all the risk-return combinations on the efficient frontier for which there is no other portfolios that have either higher returns or lower variances. We repeat this process for a second set of 100 random portfolios with 100 stocks in the S&P100 index to test the robustness of our findings.

4. Results

In order to compare the performance of S&P100 index with the performances of portfolios created using random weights, we calculate the returns, standard deviations and return-to-risk ratios for all random portfolios in two sets of weight distributions as well as for the S&P100 index. Table 1 shows the detail results.

4.1 Portfolio Return

Using equation 4 above, we estimate the returns for 100 random portfolios under two separate portfolio allocation schemes. Under the first round of portfolio allocation scheme, the daily returns for 100 random portfolios range from .0449% to .0639% with an average return of .0532%. The returns under the second portfolio scheme range from .0418% to .0645% with an average return of .0531%. As we can see, the returns under two scheme of portfolio weight are very similar. The average daily return of S&P 100 index during 2012 was about .0464% which falls at the sixth percentile among 100 different random combinations of weight in both the portfolio schemes.

4.2 Portfolio Standard Deviation

Using equation 5 above, we also calculate the portfolio standard deviations for all random portfolios under the same two weighting schemes used to calculate the portfolio returns. The portfolio standard deviations under the first round of weighting scheme range from .7880% to .8683% with an average standard deviation of .8269%. The standard deviation under second round of weighting scheme ranges from .7837% to .8773%. The average standard deviation

under the second scheme is .8308%. In case of standard deviation, we see that ranges and average values are very similar. The standard deviation of S&P100 index during 2012 was about .7859% which is the lowest among 100 different random combinations under the first scheme and second lowest under the second scheme. One observation from these results is that both average portfolio returns and average standard deviations under the both weighting schemes are very close to the realized rate of returns and standard deviations of S&P100 index during 2012.

4.3 Return-to-Risk Ratio

We also calculate the return-to-risk ratios for all portfolios as well as for the S&P100 index for 2012 for which we used the corresponding stock returns. In table 1, we present the risk (in terms of standard deviation), return and return-to-risk ratio for the top ten and bottom ten portfolios. The table shows that the top ten portfolios' return-to-risk ratios range from 7.77% to 7.06% whereas the bottom ten portfolios' ratios range from 5.75% to 5.42%. On the other hand, the same return-to-risk ratio for S&P100 index in 2012 is 5.90% which is much lower than the top ten portfolios' return-to-risk ratio. In fact, the exact place for the return-to-risk ratio of S&P100 in 2012 is at the 14th percentile level. This result basically shows that 86% of the random portfolios perform better than the S&P100 index in 2012.

The performance of the second set of random portfolios basically shows the similar results. Panel b of table 1 shows the standard deviation, return and risk-return ratios of top ten and bottom ten portfolios (in terms of risk-return ratios) and that of S&P100 index. The return-risk ratios of top ten portfolios range from 7.62% to 6.95% whereas the return-risk ratios of bottom ten portfolios range from 5.76% to 5.16%. The return-risk ratio for S&P100 index falls at around 14th percentile level. That means, 86% of the random portfolios perform better than S&P 100 index as the return-risk ratios of those portfolios are higher than that of S&P100 index.

For comparison purpose, we also present the average, minimum and maximum portfolio returns, standard deviations and return-risk ratios for both sets of random portfolios. The values between the two sets are very similar. For example, the average portfolio returns are 0.8269% and 0.8308% in the first and second schemes respectively. Similarly, the average standard deviation in the first set is 0.0532% and 0.0531% in the second set. The average return-risk ratio is .0644 in the first set and .0640 in the second set.

Table 1. Comparison of the top ten and bottom ten random portfolios based on 2012 return-to-risk ratio with that of S&P100 index

Panel a: Comparison of top ten and bottom ten portfolios of the first set of 100 portfolios with random weights.

Portfolios	Standard Deviation	Return	Return/Risk Ratio
<i>Top Ten Portfolios in terms of return-to-risk ratio:</i>			
32	0.8225%	0.0639%	0.0777
59	0.8491%	0.0634%	0.0747
57	0.8082%	0.0584%	0.0722
90	0.8368%	0.0601%	0.0719
49	0.8003%	0.0575%	0.0718
95	0.8292%	0.0591%	0.0713
46	0.8317%	0.0590%	0.0709
42	0.8380%	0.0593%	0.0708
35	0.8294%	0.0586%	0.0707
51	0.8230%	0.0581%	0.0706
<i>Bottom Ten Portfolios in terms of return-to-risk ratio:</i>			
78	0.8619%	0.0495%	0.0575
93	0.8297%	0.0475%	0.0573
60	0.8423%	0.0481%	0.0571
82	0.8320%	0.0474%	0.0569

10	0.8321%	0.0474%	0.0569
77	0.8185%	0.0461%	0.0563
33	0.8143%	0.0450%	0.0552
81	0.8184%	0.0449%	0.0549
66	0.8219%	0.0451%	0.0548
9	0.8433%	0.0457%	0.0542
<i>Descriptive Statistics</i>			
Average	0.8269%	0.0532%	0.0644
Minimum	0.7880%	0.0449%	0.0542
Maximum	0.8683%	0.0639%	0.0777
<i>Return-to-Risk ratio of S&P 100 index</i>			
2012	0.7859%	0.0464%	0.0590

Panel b: Comparison of top ten and bottom ten portfolios of the second set of 100 portfolios with random weights.

Portfolios	Standard Deviation	Return	Return/Risk Ratio
<i>Top Ten Portfolios in terms of return-to-risk ratio:</i>			
84	0.8466%	0.0645%	0.0762
50	0.8104%	0.0605%	0.0747
39	0.8500%	0.0630%	0.0741
94	0.8525%	0.0608%	0.0713
57	0.8537%	0.0605%	0.0709
62	0.8115%	0.0575%	0.0709
8	0.8006%	0.0560%	0.0699
58	0.8479%	0.0592%	0.0698
43	0.8413%	0.0585%	0.0695
44	0.8127%	0.0565%	0.0695
<i>Bottom Ten Portfolios in terms of return-to-risk ratio:</i>			
66	0.8150%	0.0469%	0.0576
37	0.8145%	0.0469%	0.0576
78	0.8258%	0.0468%	0.0567
12	0.8142%	0.0460%	0.0565
52	0.8660%	0.0485%	0.0560
95	0.8416%	0.0468%	0.0556
98	0.8040%	0.0446%	0.0555
91	0.8773%	0.0485%	0.0552
79	0.8447%	0.0461%	0.0546
6	0.8117%	0.0418%	0.0516
<i>Descriptive Statistics</i>			
Average	0.8308%	0.0531%	0.0640
Minimum	0.7837%	0.0418%	0.0516
Maximum	0.8773%	0.0645%	0.0762
<i>Return-to-Risk ratio of S&P100 index</i>			
2012	0.7859%	0.0464%	0.0590

5. Discussions and Conclusions

The above results indicate that the performance of S&P100 index in terms of return-risk ratios is not consistent with the concept of a mean-variance efficient portfolio. That means, by investing in the S&P100 index, investors may not receive the highest return for a given level of risk or the lowest risk for a given level of return. This may pose a bigger question: Does a value-weighted distribution method provide the best return-risk performance for any stock

portfolio? Since the composition of S&P100 index is based on the value-weighted portfolio distribution, it is not open to the revision of the portfolio weight that can provide the mean-variance efficient performance. Portfolio managers following this index as a passive investment strategy do not have the opportunity to achieve the optimal portfolio allocation. If they decide to use the S&P100 index as the benchmark portfolio and use the similar value-weighted distribution of funds, then their portfolio will experience the same return-risk performance. Therefore, it is important for the portfolio managers in investment firms, mutual funds, pension funds, etc. to consider the risk-return performances of their portfolios that follow this passive investment strategy and mimic any of the value-weighted stock indices to allocate their investment funds. As our results show that other random portfolio allocations provide much better risk-return performance than the performance of the S&P100 index.

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Notes

Note 1. Since our objective is to test the efficiency of S&P100 index, there should be some investment proportion in each of the stock in the index. Therefore, we set the proportion invested in each stock is a non-zero, non-negative and less than one, i.e. $0 < W_i < 1$.

Note 2. We could have easily used the price information of all stocks that were added during the year. Doing this would have changed the characteristics of the composition of the portfolio and hence it would not be representative of the actual index.