Inverse Optimization Method of Resource Allocation with Consideration of Advantage Structure

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Abstract

The research collects competence items of informal questionnaire by expert interview method, analyzes the results qualitatively and quantitatively and forms formal questionnaire. Through statistical analysis and AMOS modeling, the research obtains workers' competence model and validates competence model. An identification method of individual advantage characters according competency indicator system is built up relying on programming analysis and parameter optimization. We build the optimization model and its inverse optimization model, starting from the original optimization model, adjusting parameter value as small as possible, the conventional optimization model translated into the inverse model by the principle of duality. A calculation example is used to make sure the method is reliability and feasibility.

Keywords: competence model, advantage structure model, inverse optimization model, inverse resources allocation

1. Introduction

Competency plays an important role to promote the development of human resources performance, identifying individual advantage structure is vital to ensure person-job match scientific, advantage structure is identified by means of weight vector in optimization model, determined by making as small distance between evaluation value vector and ideal value vector as possible. MBO (Management by Objective), under the goal setting theory, based on objective management, for a given optimization problem, the inverse optimization problem is to find a minimal adjustment of the parameters (costs, capacities, etc.) of the problem such that the given solutions become optimum. Inverse optimization problem is used extensively in resources allocation, it is very useful for goal management which means when the goal is set, the resources adjustment can be done according to the goal and for the minimum change of the original model, and the classical numerical algorithm and intelligent algorithm for the inverse optimization problem have been developed.

2. Competency Model Empirical Research

The concept of "competency" was first proposed in 1970s. It is the personal condition and behavior characteristics of the outstanding person and the ordinary person in a certain field, such as motivation, knowledge, attitude, trait and so on. With the development of society, the concept of "competency" is more and more concerned and valued (Rodriguez, D., Patel, R., Bright, A., et al., 2002, pp.309-324). Before qualitatively and quantitatively analysis, the AMOS structural equation model has 142 variables, 69 observable variables, 73 unobserved variables, 73 exogenous variables, 69 endogenous variable. The competency path model is showed in Table 1.

Table 1. Path coefficient of regression analysis

Path			Estimate	S.E.	C.R.	P	Label	Std.
a1	<---	F1	1.000					.696
a2	<---	F1	.825	.069	12.031	***	par_1	.690
a3	<---	F1	.819	.067	12.256	***	par_2	.703
a4	<---	F1	.752	.064	11.666	***	par_3	.669
a5	<---	F1	1.067	.075	14.211	***	par_4	.823
a6	<---	F1	1.079	.078	13.889	***	par_5	.804
a7	<---	F1	.951	.074	12.925	***	par_6	.746
a8	<---	F1	.902	.069	13.013	***	par_7	.752
a9	<---	F1	1.128	.080	14.070	***	par_8	.814
a10	<---	F1	1.036	.077	13.380	***	par_9	.773
a11	<---	F1	1.079	.075	14.324	***	par_10	.829
a12	<---	F1	1.004	.074	13.560	***	par_11	.784
a13	<---	F1	1.106	.082	13.421	***	par_12	.775
a14	<---	F1	.965	.073	13.244	***	par_13	.765
a15	<---	F1	.970	.071	13.704	***	par_14	.792
a16	<---	F1	.994	.072	13.841	***	par_15	.801
a17	<---	F1	.906	.066	13.633	***	par_16	.788
a18	<---	F1	.931	.070	13.323	***	par_17	.770
a19	<---	F1	1.003	.073	13.660	***	par_18	.790
a20	<---	F1	.995	.070	14.149	***	par_19	.819
a21	<---	F1	.762	.061	12.528	***	par_20	.722
a22	<---	F1	.929	.072	12.935	***	par_21	.747
a23	<---	F1	.887	.069	12.910	***	par_22	.745
a24	<---	F1	.984	.069	14.188	***	par_23	.821
a25	<---	F1	.933	.067	14.006	***	par_24	.810
a26	<---	F1	.898	.065	13.859	***	par_25	.802
a27	<---	F1	1.051	.074	14.227	***	par_26	.823
a28	<---	F1	1.004	.073	13.843	***	par_27	.800
a29	<---	F1	.952	.070	13.580	***	par_28	.784
a30	<---	F1	.977	.069	14.075	***	par_29	.814
a31	<---	F1	1.015	.075	13.600	***	par_30	.786
a32	<---	F1	1.050	.076	13.901	***	par_31	.804
a33	<---	F1	.954	.070	13.581	***	par_32	.784
a34	<---	F1	.938	.069	13.504	***	par_33	.780

Path			Estimate	S.E.	C.R.	P	Label	Std.
b15	<---	F2	.249	.029	8.570	***	par_34	.757
b14	<---	F2	.269	.031	8.613	***	par_35	.766
b13	<---	F2	.270	.031	8.636	***	par_36	.771
b12	<---	F2	.273	.032	8.532	***	par_37	.749
b11	<---	F2	.297	.034	8.708	***	par_38	.786
b10	<---	F2	.282	.032	8.690	***	par_39	.782
b9	<---	F2	.284	.033	8.720	***	par_40	.788
b8	<---	F2	.265	.031	8.571	***	par_41	.756
b7	<---	F2	.293	.034	8.669	***	par_42	.777
b6	<---	F2	.266	.031	8.609	***	par_43	.764
b5	<---	F2	.260	.030	8.567	***	par_44	.755
b4	<---	F2	.300	.035	8.553	***	par_45	.752
b3	<---	F2	.304	.035	8.689	***	par_46	.781
b2	<---	F2	.333	.038	8.857	***	par_47	.820
b1	<---	F2	.330	.038	8.781	***	par_48	.802
c1	<---	F3	1.000					.645
c2	<---	F3	1.497	.124	12.035	***	par_49	.775
c3	<---	F3	1.279	.112	11.406	***	par_50	.725
c4	<---	F3	1.115	.101	11.005	***	par_51	.695
c5	<---	F3	1.630	.129	12.651	***	par_52	.826
c6	<---	F3	1.684	.132	12.722	***	par_53	.832
c7	<---	F3	1.572	.124	12.665	***	par_54	.827
c8	<---	F3	1.499	.125	12.025	***	par_55	.774
c9	<---	F3	1.440	.117	12.294	***	par_56	.796
c10	<---	F3	1.459	.119	12.276	***	par_57	.795
c11	<---	F3	1.233	.108	11.396	***	par_58	.725
c12	<---	F3	1.089	.102	10.697	***	par_59	.672
c13	<---	F3	1.606	.123	13.022	***	par_60	.857
c14	<---	F3	1.444	.118	12.256	***	par_61	.793
c15	<---	F3	1.045	.097	10.733	***	par_62	.674
d1	<---	F4	1.000					.876
d2	<---	F4	1.006	.044	23.080	***	par_63	.898
d3	<---	F4	1.030	.043	23.971	***	par_64	.914
d4	<---	F4	.971	.046	21.220	***	par_65	.863
d5	<---	F4	.839	.045	18.768	***	par_66	.809

The correlation coefficient between the latent variables of competency in structural model is shown in Table 2.

Table 2. The correlation coefficient between the latent variables

			covariance	S.E.	C.R.	P	Label	coefficient
successful general characteristics	<-->	knowledge of automatic production line	1.000					.834
risk control ability	<-->	successful general characteristics	.290	.034	8.645	***	par_67	.749
successful general characteristics	<-->	professional quality	.187	.024	7.905	***	par_68	.820
knowledge of automatic production line	<-->	professional quality	.574	.057	9.998	***	par_69	.800
risk control ability	<-->	knowledge of automatic production line	.777	.071	10.941	***	par_70	.637
risk control ability	<-->	professional quality	.157	.020	7.859	***	par_71	.676

Fitting degree evaluation if competency in structural model is shown in Table 3.

Table 3. Fit index of competency structural model

Model	NFI	RFI	IFI	TLI	CFI	RMSEA	PCLOSE	NPAR	CMIN	DF	P	CMIN/DF
default	.626	.602	.680	.658	.678	.097	.000	213	10623.321	2271	0	4.678
saturated	1.0		1.00		1.0	.167	.000	2484	.000	0		
independent	.0	.000	.000	.000	.000			69	28389.69	2415	0	11.756

To sum up, the competency indicator system contains the following indicators: knowledge of automatic production line, successful general characteristics, professional quality.

3. Advantage Structure Identification Model

In fact, evaluations always have guiding functions, which usually contains three major aspects: the first one is determined by each indicator of evaluation index system. The second one is determined by the result of the ideal value. This function is only in object programming evaluation methods. The third one is expressed by the vector of value parameter, the vector of value parameter could be used to express their advantages and weaknesses.

The advantage structure identification model should be comparable to the ideal condition which provides a clear guidance in the behavior adjusting process. Since the limitation of detection technology, level of many indexes can not be evaluated straightly (such as competency). In this case, using collected information, which can be obtained by questionnaire through choosing a number of respondents who have contacted with the evaluated object, inviting them to assess object's behavior level on each item. Since one respondent can not acquaint in all respects, it needs to get the evaluation information by group decision making. Although valuations are usually influenced by certain environment with a psychological tendency to express one's talent and public image. This tendency moves valuations to cover up his emotional preferences and display "justness". In other words, even if the existence of subjective influence on the evaluation such as psychological effect, fairly reveal the object's strength and weakness. So this paper uses average score as indicator value.

Moreover, each decision maker has his unique angle which we can hardly judge whether it is right or wrong then and there, and the evaluation object hopes to be evaluated by maximization its social contribution or work efforts and obtained achievements from standing out their individual advantage characteristics. So it is necessary to find out the

object's quantitative advantage structure based on observers' evaluation, and get one aggregate-value.

Among the existing utility functions, the object oriented utility function is object programming evaluations. There are p indicators, Person i's advantage structure identification model is:

$$d^2(x_i - x^*) = \sum_{j=1}^{p} w_j^2 (x_j^* - x_{ij})^2 \tag{1}$$

where $x^* = (x_1^*, x_2^*, \cdots, x_p^*)^\tau$ represents the ideal value of each indicator, $x_i = (x_{i1}, x_{i2}, \cdots, x_{ip})^\tau$ refers to respondent evaluation value on the object of each indicator, $w = (w_1, w_2, \cdots, w_p)^\tau$ is the vector of value parameter (weight vector). Actually, function (1) gives the distance (square) between the evaluation value and ideal value.

In order to identify object's individual advantage structure from the point of view of each respondent, to determine the vector weight by making the object most propitious. It can be carried out by model (2) (Zhang, L., Hou, X., & Xi, F., 2013, pp.160-164):

$$\min d_i^2\left(x_i, x^*\right) = \sum_{j=1}^{p} w_{ij}^2 \left(x_j^* - x_{ij}\right)^2$$

$$s.t: \quad \sum_{j=1}^{p} w_{ij} = 1 \tag{2}$$

$$w_{ij} \geq 0 \quad j=1,2,\cdots p; i=1,2,\cdots n$$

The objective function in model (2) is continuous, and the feasible region of constrained conditions is bounded, so the most optimal solutions of model (2) exist. The solution set of function(2) is as following:(i)If there is at least one evaluation value equal to ideal value, then the sum of the components which reach the ideal value is 1, other index weights are all 0; (ii)If there is no evaluation value reaching ideal value, the weight can be obtained by (Zhang, L., 2011, pp.102-105):

$$w_{ij}^* = \frac{\lambda_i^*}{(x_{ij}^* - x_{ij})^2} \tag{3}$$

$$\lambda_i^* = \frac{1}{\displaystyle\sum_{j=1}^{p} \frac{1}{(x_{ij}^* - x_{ij})^2}}, i = 1,2,...,n, j = 1,2,...,p \tag{4}$$

The comprehensive evaluation method is:

$$c_i = \sum_{j=1}^{p} w_{ij} x_{ij} \tag{5}$$

4. Optimization Model and Inverse Optimization Model

The optimization model is as following.

$$max(min)f = \sum_{j=1}^{p} w_{1j} x_{1j} y_1 + \sum_{j=1}^{p} w_{2j} x_{2j} y_2 + \cdots + \sum_{j=1}^{p} w_{nj} x_{nj} y_n$$

$$s.t. \begin{cases} a_{11} y_1 + a_{12} y_2 + \cdots + a_{1n} y_n \leq (=,\geq) = b_1 \\ a_{21} y_1 + a_{22} y_2 + \cdots + a_{2n} y_n \leq (=,\geq) = b_2 \\ \quad \cdots \\ a_{m1} y_1 + a_{m2} y_2 + \cdots + a_{mn} y_n \leq (=,\geq) = b_m \\ y_j \geq 0 (j = 1,2,...,n) \end{cases} \tag{6}$$

When seeing an optimization problem, we usually assume that the parameters such as costs, capacities, efficiency etc., are known and that we are interested in finding an optimal solution. However, in practice, it may happen that we only know estimates for the parameters. Additionally, we might know that certain are optimal from observations. The idea of inverse optimization is to find values of the parameters which make the known solutions optimum and which differ from the given estimates as little as possible (Heuberger, C., 2004, pp.329-361).

Inverse optimization based on the linear optimization model. Making a linear programming model of a given solution to realize optimization by adjusting the coefficient vector. According to the feasible solution of the optimal solution, the optimal solution of the plan can be achieved by adjusting the cost coefficient or the resource constraint. The method based on linear inverse problem enables companies to be more flexible in corresponding demand.

The optimization model is:

$$\min z = c^T y$$
$$(s.t.) \begin{cases} Ay \geq b \\ \\ y \geq 0 \end{cases} \tag{7}$$

A kind of dual problems of linear programming is established as the following.

$$\max z = \omega b$$
$$(s.t.) \begin{cases} \omega A \leq c \\ \\ \omega \geq 0 \end{cases} \tag{8}$$

Given model (7) a feasible solution y^0, make the inverse optimization conversion, make y^0 the optimal solution,

according to the dual theory, model (8) has the optimal solution ω^0. Then:

$$\omega^0 (Ay^0 - b) = 0 \tag{9}$$

$$(c - \omega^0 A) y^0 = 0 \tag{10}$$

$$\min \left\| \overline{c} - c \right\|$$
$$s.t. \begin{cases} \omega^0 a_j \leq \overline{c}_j, y_j = 0 \\ \omega^0 a_j = \overline{c}_j, y_j \rangle 0 \end{cases} \tag{11}$$

$\overline{c}_j = c_j + \varepsilon_j - \beta_j$, total cost change is: $\varepsilon - \beta$, the inverse optimization can be changed as:

$$\min \left\| \varepsilon + \beta \right\|$$
$$s.t. \begin{cases} \omega^0 a_j - \varepsilon_j + \beta_j \leq c_j, y_j = 0 \\ \omega^0 a_j - \varepsilon_j + \beta_j = c_j, y_j \rangle 0 \\ \varepsilon_j, \beta_j \geq 0 \end{cases} \tag{12}$$

Given model (7) a non-feasible solution y^0, make the inverse optimization conversion, make y^0 the optimal

solution, according to the dual theory, model (8) has the optimal solution ω^0. Then:

$$\min\left\|\bar{b} - b\right\|$$
$$s.t.\begin{cases} a_i y^0 - b_i \geq 0, \omega_i = 0; \\ a_i y^0 - b_i = 0, \omega_i > 0 \end{cases} \tag{13}$$

Let $\bar{b}_i = b_i + \alpha_i - \theta_i$, i=1,2, …, m, $\alpha_i \geq 0, \theta_i \geq 0$, then:

$$\min\left\|\alpha + \beta\right\|$$
$$s.t.\begin{cases} a_i y^0 - \alpha_i + \beta_i \geq b_i, \omega_i = 0 \\ a_i y^0 - \alpha_i + \beta_i = b_i, \omega_i = 0 \\ \alpha_i \geq 0 \\ \beta_i \geq 0 \end{cases} \tag{14}$$

5. Calculation Example Analysis

$$\min z = 15 x_1 + 24 x_2 + 5 x_3$$
$$(s.t.)\begin{cases} 6x_2 + x_3 \geq 2 \\ 5x_1 + 2x_2 + x_3 \geq 1 \\ x_1, x_2, x_3 \geq 0 \end{cases} \tag{15}$$

Applying simplex algorithm, the optimal solution of the optimization problem is $y^0 = (0, 1/4, 1/2)$.

We want to make a non-feasible solution $x^0 = (1/5, 1/5, 0)$ the optimal solution for the model(7), we need to change b, so the mathematical model is:

$$\min u = \alpha_2 + \beta_2 + \alpha_1 + \beta_1$$
$$(s.t.)\begin{cases} \alpha_1 - \beta_1 \geq \dfrac{4}{5} \\ \alpha_2 - \beta_2 \geq -\dfrac{2}{5} \\ \alpha_1, \beta_1, \alpha_2, \beta_2 \geq 0 \end{cases} \tag{16}$$

We use MATLAB to solve this problem, the optimal solution is (0,0.4,0,0), and we change b_1 from 2 to 1.6, others do not need to change.

6. Conclusion

The arrival of intelligent era will not only intelligent applied to all areas of life, but also for future workers put forward a severe demand. In the future, especially in the industries with high intelligence penetration, people need to evaluate their work with their own new ideas. Because some indicators measure inconvenience, the details of the division of competencies are built. Based on the existing performance evaluation methods, this paper gives a method of identifying the advantage structure of individuals.

This paper incorporates the academic thinking of inverse optimization, not only puts psychology and behavior into optimization model, but also data mines strengths characteristics under the psychology and behavior data, and find a new way to introducing the strengths characteristics into optimization model.

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