Modified VOGEL Method to Find Initial Basic Feasible Solution (IBFS) Introducing a New Methodology to Find Best IBFS

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Abstract

This research presents transportation modeling approach to solve transportation problems that flow of items from a certain number of suppliers with various production capacities to a certain points of destination. The aim of the paper is to achieve minimum cost of transportation flow for an initial basic feasible solution (IBFS) which consequently will be closer to the optimal solution of the model. Classical transportation methods for distribution are discussed in details and their IBFS are calculated and compared to the results of newly introduced methodology. The results show that the new method has an advantage over the classical transportation methods through achieving a competitive cost for the IBFS.

Keywords: Transportation, Models, IBFS, Optimal, Distribution methods

1. Introduction

As the economic slowdown has hit most of the business and production sectors, thus cost minimization has become a very important issue in today's business and in order for companies to survive, cost rationalization has become priority task for them (Edu, Bernard Enya and Esang, Atim E. and Otonkue, Agba D. O., 2009) .Transportation modeling using Linear programming technique is one of those ways that can help to find an optimal solution to companies and save the costs in transportation activities. The philosophy of these methods was originally established in 1941 but the real push through was made by the American mathematician Dantzig in 1953. The most recognized methods to solve transportation problems such as North-West-Corner, Least Cost and Vogel have attracted many scientists in this area such as Napeir , Golver, and Karney (Glover, F., D. Karney, and D. Klingman, and A. Napier 1974) and later Dantzig ,Kantorovich, Hitchcock, and Koopmans (Dantzig ,1963). These scientists have examined methods and put them in practice, and combined them with software development process in order to reach a fast and accurate optimal solution to various transportation problems.

Transportation models are a special class of linear programs that deal with shipping a commodity from sources (e.g., factories) to destinations (e.g., warehouses). The objective of the model is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. The model assumes that the shipping cost is proportional to the number of units shipped on a given route. In general, the transportation model can be extended to other areas of operation, such as inventory control, employment scheduling, and personnel assignment (Hamdy A Taha, 2002).

2. Transportation Problem Model

A transportation problem basically deals with the problem, which aims to find the best way to fulfill the demand of n demand points using the capacities of m supply points. While trying to find the best way, generally a variable cost of shipping the product from one supply point to a demand point should be taken into consideration. Figure (1) depicts an example on network of transportation problem when C_{ij} represent the cost involved in transporting one item from supply area i to demand area j.

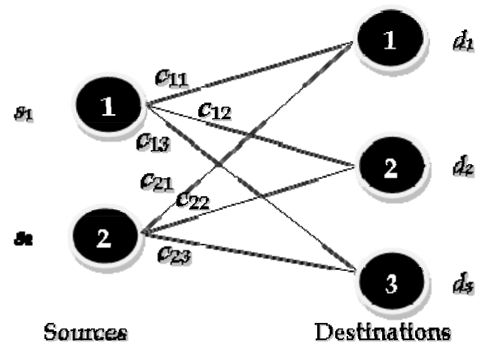

Figure 1. represents Transportation network

In general, a transportation problem can be stated mathematically as follows:

		Cost per Unit Distributed				
		Destination				
		1	2	...	n	Supply
Source	1	c_{11}	c_{12}	...	c_{1n}	s_1
	2	c_{21}	c_{22}	...	c_{2n}	s_2
	⋮					⋮
	m	c_{m1}	c_{m2}	...	c_{mn}	s_m
Demand		d_1	d_2	...	d_n	

Let *m* be a set of supply sources from which a good is shipped. Supply source *i* can supply at most s_i units.

Let *n* be a set of demand destinations to which the good is shipped. Demand destination *j* must receive at least d_i units of the shipped good.

Each unit produced at supply point *i* and shipped to demand point *j* incurs a variable cost of c_{ij}.

x_{ij} = number of units shipped from supply point *i* to demand point j

$$\text{Min (cost)} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}X_{ij}$$

s.t.

$$\sum_{j=1}^{n} X_{ij} \le s_i (i = 1,2,..., m)$$

$$\sum_{i=1}^{m} X_{ij} = d_j (j = 1,2,...,n)$$

$$X_{ij} \ge 0 (i = 1,2,..., m; j = 1,2,..., n)$$

If $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$, then total supply equals total demand, and the problem is said to be a *balanced transportation problem*.

Transportation Problem is a normal linear-programming problem and can be solved by direct application of the simplex method. However, because of its very special mathematical structure, it can be made very efficient in terms of how to calculate the necessary simplex-method steps through entering a variable in the basis, and also making a variable leaves the basis to reach the optimality conditions (G.B.Dantzig , 1951). Dantzig has adapted the simplex method to the transportation problem by using what is referred to as the MODI method (modified Distribution Method)(Anderson, Sweeney, Williams, Wisniewski. 2009). Later Charnes and Cooper (CHARNES A. and W. W. COOPER (1954)) developed the stepping-stone method (SSM) which provides an alternative way of determining the simplex method optimality condition. Kirca and Satir (Kirca O. and Satir A ,1990) describe a heuristic method for obtaining an initial solution for the transportation problem based on the total opportunity cost concept.

3. Transportation Algorithm Mechanism

According to Jay Heizer and Barry Render, "Transportation modeling is an iterative procedure for solving problems that involve minimizing the cost of shipping products from a series of sources to a series of destinations"(Jay Heizer, Barry Render, 2004). Heizer in his book explained the steps for solving the problem as "Based on theory, after all needed data were arranged in tabular form, the next step of the technique is to establish an initial feasible solution to the problem". In accordance to the transportation problem, the following steps are to be followed:

Step 1: Find IBFS by using one of the transportation modeling methods such as Northwest corner, Lowest-Cost and Vogel's Approximation method. These methods will be explained in more details later on. If the initial feasible solution is degenerate with less than $(m+n-1)$ occupied cells, add one or more artificial occupied cell or cells so that $m+n-1$ cells exist in locations that enable the use of the MODI method of testing optimality. Make sure that the transportation problem is balanced meaning that the total supply quantity must equal the total demand quantity.

Step 2: Use the *MODI* method to compute row indexes, U_i, and column indexes, V_j.

Step 3: Compute the net evaluation index $e_{ij}=C_{ij}-U_i-V_j$ for each unoccupied cell.

Step 4: If $e_{ij}\geq 0$ for all unoccupied cells, stop; you have reached the minimum cost solution. Otherwise, proceed to step 5.

Step **5:** Identify the unoccupied cell with the smallest (most negative) net evaluation index and select it as the incoming cell.

Step 6: Find the path associated with the incoming cell. Label each cell on the path whose flow will increase with a plus-sign and each cell whose flow will decrease with a minus sign.

Step 7: Choose as the outgoing cell the minus-sign cell on the path with the smallest flow. If there is a tie, chose any one of the tied cells. The tied cells that are not chosen will be artificially occupied with a flow of zero at the next iteration.

Step 8: Allocate to the incoming route or cell the amount of flow currently given to the outgoing cell; make the appropriate adjustments to all cells on the stepping-stone path, and continue with step 2.

4. Purpose of the study

The main target of this paper is not to find the optimal solution for the transportation problem. Instead, the paper presents an in-depth computational comparison of the basic feasible solution algorithms for solving transportation problems using the classical methods and the proposed method. The introduced methodology develops a solution procedure that can capture the best IBFS to the problem which consequently may reach the optimal solution faster than any other solution. Therefore, the methodology in this paper is to produce the basic feasible solutions using the classical transportation modeling methods followed by the introduced new methodology. Based on calculations and results of different methods and approaches to the same transportation problem, using different cases, it can be proved that the new methodology can capture better IBFS compared to other classical methods. That is the comparison of these methods will show that the new methodology can have an advantage over the three classical methods (i.e. North West Corner Method (NWCM), Least Cost Method (LCM), and Vogel Approximation Method (VAM). The paper investigates possible ways of minimizing the cost of transportation using manual calculation and additionally TORA Optimization System Windows-based software (Hamdy A Taha. 2003). These two tools will help to understand the details of the transportation algorithm by describing all steps involved.

5. Obtaining an Initial Basic Feasible Solution in Transportation Algorithm:

Obtaining a basic primal start for the constrained transportation problem is easily handled by extending the pure start algorithms that exist for the transportation problem.

A variety of start algorithms exist for obtaining a basic feasible start for the pure transportation problem.

Primal start methods include the Northwest Corner rule (NWCM), Lowest Cost Method (LCM), and VOGEL approximation method (VAM). Brief explanations of the three methods are presented below.

The Northwest Corner Method: With the northwest corner method, an initial allocation is made to the cell in the upper left-hand corner of the tableau (i.e., the "Northwest Corner"). The amount allocated is the most possible, subject to the supply and demand constraints for that cell. Then allocations to adjacent feasible cells are made as follows:

1) Select the upper left (north-west) cell of the transportation matrix and allocate the maximum possible value to X_{11} which is equal to min (a_1, b_1).

2) If allocation made is equal to the supply available at the first source (a_1 in first row), then move vertically down to the cell (2, 1). If allocation made is equal to demand of the first destination (b_1 in first column), then move horizontally to the cell (1, 2).

   If a_1=b_1, then allocate X_{11}= a_1 or b_1 and move to cell (2, 2).

3) Continue the process until an allocation is made in the south-east corner cell of the transportation table.

The Minimum Cost Method: With the minimum cell cost method, the basic logic is to allocate to the cells with the lowest costs. The initial allocation is made to the cell in the tableau having the lowest cost. The following step can describe the method:

1) Select the cell having lowest unit cost in the entire table and allocate the minimum of supply or demand values in that cell.

2) Then eliminate the row or column in which supply or demand is exhausted. If both the supply and demand values are same, then both of the row or column can be eliminated.

   In case, the smallest unit cost is not unique, then select the cell where maximum allocation can be made.

3) Repeat the process with next lowest unit cost and continue until the entire available supply at various sources and demand at various destinations is satisfied.

VOGEL Approximation Method (VAM): Vogel method for a balanced transportation problem starts with the calculation of penalties (the difference between the second minimum and the first minimum costs) for all rows and columns. Then it allocates as many units as possible to the least-cost cell in the row or column having maximum penalty.

The following steps explain the methodology of the method (Ramakrishna C.S. 1988):

1) Calculate penalty for each row and column by taking the difference between the two smallest unit costs. This penalty or extra cost has to be paid if one fails to allocate the minimum unit transportation cost.

2) Select the row or column with the highest penalty and select the minimum unit cost of that row or column. Then, allocate the minimum of supply or demand values in that cell. If there is a tie, then select the cell where maximum allocation could be made.

3) Adjust the supply and demand and eliminate the satisfied row or column. If a row and column are satisfied simultaneously, only one of them is eliminated and the other is assigned a zero value. Any row or column having zero supply or demand cannot be used in calculating future penalties.

4) Repeat the process until all the supply sources and demand destinations are satisfied.

Then the allocated row/column is deleted, penalties are revised and the procedure repeated successively until all units are supplied.

6. The Proposed Methodology for obtaining IBFS

As mentioned earlier, that primary aim of this paper is to find BFS that can compete with other existing classical methods in producing BFS and its corresponding cost value. The prototype methodology can give even better results than any current classical methods available in the Operations Research literature (See examples of the new methodology and calculations of IBFS results).

One of the main reasons to develop this method was to improve VOGEL method which is considered as one of the best methods to obtain lowest IBFS cost. But when the calculation of VOGEL method's penalties is considered, it can be noticed that each penalty value calculation for any column or row in the transportation problem table depends solely on the difference between the lowest two transportation costs in that column or row of the transportation cost table regardless of the other costs. This could mask the real variation that may exist among the costs in any row or column and may give misleading penalties. The VAM method according to its calculations of penalties cannot distinguish the altitude of the differences in all columns and rows as it depends only on smallest two costs in each row and column. Therefore, penalty cannot reflect the size of numbers in any row or column. The idea of the research is to work on the calculation of any penalty depending on all values in the entire row or column.

From this pivot point I decided to calculate the penalties of VAM method by computing the "Median Cost" of each column and row as the Median can be considered a very a good estimator of the cost in any particular row or column. Statistically, "Median" is considered best indicator when we have extreme values in the data, and this is what we expect to face in most of transportation problem data.

In order to find the penalties of the transportation table via Median calculations, a number of steps are to be followed to reach the targeted IBFS. These steps can be summarized as follows:

1- Make sure that the problem is balanced, that is the total supply equals the total demand quantities. If the problem is not balanced we can make it balanced by adding either dummy row or dummy column. Thus, the requirement is: $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$

2- Create additional (n+m-3) columns adjacent to transportation problem table columns. These columns are needed to calculate the Median cost of transportation for each row (Origin).

3- Create additional (n+m-3) rows below the transportation problem table rows. These rows are needed to calculate the Median cost of transportation for each column (Destination).

4- Select the highest Median cost among all median values of columns and rows.

5- Whether the highest median cost is belongs to column or row, find, what is the lowest cost cell (ij) (best) inside the corresponding column or row.

6- Satisfy the cheapest best cell (ij) with quantity according to available supply of the row (i) and the amount of demand in column (j).

7- Discard the column (j) or row (i) (or both) from future calculation, if the left over quantity in column (j) or row (i) (or both) after satisfying (ij) cell will became zero.

8- Go back to steps 2 to 7 until all supply and demand quantities are exhausted.

9- Calculate the transportation cost of the initial basic solution.

10- Normally, we carry the test of optimality after last step. If the basic feasible solution satisfies the optimality condition, we stop and optimal solution is reached. Otherwise we carry on until we reach the optimal solution.

The last step (10) above is not part of research goals, as we are interested in finding only the basic feasible solution.

7. Numerical Examples

In order to gauge the effectiveness of the new methodology, several examples with different number of rows and columns are given as follows:

<u>Example 1: Transportation problem 4x4</u>:

Cells in tables below with gray color represent the rote from supply area to demand area; whereas the cells with black color represent the penalty value calculated using the new methodology. Each example has been solved using four methods of transportation (i.e Northwest Corner Method-NWC, Least Cost Method-LCM , Vogel Meshed , and the new introduced method), as it is shown below:

4x4	NWC Method IBFS Cost =3465			
	40	65	75	120
90	(14) 40	(9) 50	(15)	(10)
80	(10)	(16) 15	(13) 65	(20)
70	(9)	(5)	(11) 10	(12) 60
60	(18)	(8)	(6)	(9) 60

4x4	Least Cost Method IBFS Cost =2775			
	40	65	75	120
90	14	9	15	10 90
80	10 35	16	13 15	20 30
70	9 5	5 65	11	12
60	18	8	6 60	9

4x4	VOGEL Method IBFS Cost =2740			
	40	65	75	120
90	14	9	15	10 90
80	10 40	16	13 15	20 25
70	9	5 65	11	12 5
60	18	8	6 60	9

New Methodology

4x4	New Method IBFS Cost =2650				1st	2nd	3rd	4th	5th
	40	65	75	120					
90	14	9	15	10 90	12	10	10	12.5	-
80	10 40	16	13 40	20	14.5	16	-	-	-
70	9	5 65	11 5	12	10	11	11	11.5	11.5
60	18	8	6 30	9 30	8.5	8	8	7.5	7.5
1st	12	8.5	12	11					
2nd	-	8	11	10					
3rd	-	-	11	10					
4th	-	-	11	10					
5th	-	-	8.5	10.5					

Conclusion on (4x4) Problem:

1- Northwest Corner Method Cost = 3465 (Using TORA Software)
2- Least Cost Method Cost =2775 (Using TORA Software)
3- VOGEL Method Cost =2740 (Using TORA Software)
4- New Methodology Cost = 2650 (Best IFBS)

Example 2: Transportation problem 5x4:

5x4	NWC – IBFS Cost=16500							
	500		600		200		200	
200	200	10		20	0	5		7
300	300	13		9		12		8
200	0	4	200	15		7		9
400		14	400	7		1		1
400		3	0	12	200	5	200	19

5x4	LCM – IBFS Cost=10200							
	500		600		200		200	
200		10	200	20	0	5		7
300		13	300	9		12		8
200	100	4	100	15		7		9
400		14		7	200	1	200	1
400	400	3		12		5		19

5x4	VOGEL – IBFS Cost=9800			
	500	600	200	200
200	10 200	20	5 0	7
300	13	9 300	12	8 100
200	4 200	15	7	9
400	14	7	1 200	1 200
400	3 100	12 300	5	19

5x4	New Methodology IBFS Cost =8800				1st	2nd	3rd	4th	5th	6th
	500	600	200	200						
200	10	20	100 5	100 7	8.5	8.5	7	7	7.5	-
300	13	200 9	12	100 8	10.5	10.5	12	-	-	-
200	200 4	15	7	9	8	8	7	7	5.5	5.5
400	14	400 7	1	1	4	-	-	-	-	-
400	300 3	12	100 5	19	8.5	8.5	5	5	4	4
1st	10	12	5	8						
2nd	7	13.5	6	8.5						
3rd	7	-	6	8.5						
4th	4	-	5	9						
5th	4	-	5	-						
6th	3.5	-	6	-						

Conclusion on (5x4) Problem:

1- Northwest Corner Method Cost = 16500 (Using TORA Software)
2- Least Cost Method Cost = 10200 (Using TORA Software)
3- VOGEL Method Cost = 9800 (Using TORA Software)
4- New Methodology Cost = 8800 (Best IFBS)

Example 3: Transportation problem 6x6:

6x6	NWCM – IBFS Cost=11100					
	300	500	700	300	250	250
400	5 300	1 100	2	3	4	7
500	7	2 400	3 100	1	5	6
300	9	1	9 300	5	2	3
150	6	5	8 150	4	1	4
600	8	7	11 150	6 300	4 150	5
350	2	5	7	5	2 100	1 250

6x6	LCM – IBFS Cost =9100					
	300	500	700	300	250	250
400	5	1 400	2	3	4	7
500	7	2	3 200	1 300	5	6
300	9	1 100	9 100	5	2 100	3
150	6	5	8	4	1 150	4
600	8 200	7	11 400	6	4	5
350	2 100	5	7	5	2	1 250

6x6	VOGEL – IBFS Cost =7100					
	300	500	700	300	250	250
400	5	1	2; 400	3	4	7
500	7	2	3; 200	1; 300	5	6
300	9	1; 200	9	5	2; 100	3
150	6	5	8	4	1; 150	4
600	8	7; 300	11; 100	6	4	5; 200
350	2; 300	5	7	5	2	1; 50

6x6	New Methodology IBFS Cost =6650														
	300	500	700	300	250	250	1st	2nd	3rd	4th	5th	6th	7th	**8th**	**9th**
400	5	1	2; 400	3	4	7	3.5	0	0	0	0	0	0	**0**	**0**
500	7	2; 150	3; 300	1; 50	5	6	4	4	5	3.5	2	1.5	1.5	**1.5**	**1.5**
300	9	1; 300	9	5	2	3	4	4	3	2.5	3	3	3	**3**	**3**
150	6	5	8	4; 150	1	4	4.5	4.5	4	4	4	4.5	4.5	**4.5**	**0**
600	8	7	11	6; 100	4; 250	5; 250	6.5	6.5	6	5.5	6	6.5	0	**0**	**0**
350	2; 300	5; 50	7	5	2	1	3.5	3.5	2	3.5	5	5	5	**0**	**0**
1st	6.5	3.5	7.5	4.5	3	4.5									
2nd	7	5	8	5	2	4									
3rd	7	5	0	5	2	4									
4th	0	5	0	5	2	4									
5th	0	5	0	5	0	4									
6th	0	5	0	5	0	0									
7th	0	3.5	0	4.5	0	0									
8th	0	2	0	4	0	0									
9th	0	1.5	0	3	0	0									

Conclusion on (5x4) Problem:

1- Northwest Corner Method Cost =11100 (Using TORA Software)

2- Least Cost Method Cost = 9100 (Using TORA Software)

3- VOGEL Method Cost = 7100 (Using TORA Software**)**

4- New Methodology Cost = 6650 (Best IFBS)

Example 4: (10x10) Transportation Problem

10x10	NWCM – IBFS Cost=110500									
	1000	500	200	300	300	600	100	500	400	200
500	10 500	30	50	70	20	40	20	35	40	15
300	20 300	30	35	45	65	55	25	15	10	50
700	40 200	10 500	20	15	30	25	45	55	60	25
250	60	15 0	30 200	20 50	60	45	35	40	25	10
750	20	20	35	25 250	55 300	40 200	30	50	20	30
700	10	25	25	30	40	35 400	25 100	45 200	30	40
500	20	30	15	40	35	15	20	40 300	10 200	45
100	25	35	10	50	25	20	15	20	25 100	55
150	30	45	25	35	15	30	10	30	30 100	30 50
150	55	30	45	15	20	10	20	15	25	15 150

10x10	LCM – IBFS Cost=79750									
	1000	500	200	300	300	600	100	500	400	200
500	10 (500)	30	50	70	20	40	20	35	40	15
300	20	30	35	45	65	55	25	15	10 (300)	50
700	40	10 (500)	20	15 (200)	30	25	45	55	60	25
250	60	15	30	20 (50)	60	45	35	40	25	10 (200)
750	20	20	35	25 (50)	55 (200)	40	30	50 (500)	20	30
700	10 (500)	25	25	30	40 (50)	35 (150)	25	45	30	40
500	20	30	15 (100)	40	35	15 (300)	20	40	10 (100)	45
100	25	35	10 (100)	50	25	20	15	20	25	55
150	30	45	25	35	15 (50)	30	10 (100)	30	30	30
150	55	30	45	15	20	10 (150)	20	15	25	15

10x10	VOGEL – IBFS Cost=77000									
	1000	500	200	300	300	600	100	500	400	200
500	10 300	30	50	70	20	40	20	35	40	15 200
300	20	30	35	45	65	55	25	15	10 300	50
700	40	10 500	20 150	15 50	30	25	45	55	60	25
250	60	15	30	20 250	60	45	35	40	25	10 0
750	20	20	35	25	55	40 200	30 100	50 450	20	30
700	10 700	25	25	30	40	35	25	45	30	40
500	20	30	15	40	35	15 400	20	40	10 100	45
100	25	35	10 50	50	25	20	15	20 50	25	55
150	30	45	25	35	15 150	30	10	30	30	30
150	55	30	45	15	20 150	10	20	15	25	15

Conclusion on (10x10) Problem:

1- Northwest Corner Method Cost =110500 (Using TORA Software)

2- Least Cost Method Cost = 79750 (Using TORA Software)

3- VOGEL Method Cost = 77000 (Using TORA Software**)**

4- New Methodology Cost = 62500 (Best IFBS) (See appendex-1 for detailed calculations)

8. Conclusion:

The research was conducted in order to analyze the possibility of improving the different transportation modeling methods which are used in most operations research books and restructures, in terms of cost function by finding best IBFS for transportation routes from different production sources to different points of destination. The software of TORA (Hamdi Taha's Software) application was used in order to show the results of IBFS of the classical modeling methods (NWCM, LCM, and VAM) for the transportation route of an organization. Whereas, the calculations of the new methodology solutions were made manually using Excel software to help in calculating the median cost for each row and column of transportation problem. All steps and details of calculation, using manual calculations and TORA software were presented in the analysis. The study is important, as the minimization of transportation costs and optimizing of the transportation processes may help to improve the organization's position in the market and increase its profitability by reduction of expenses on transportation. The analysis of the new methodology has showed the superiority in finding the IBFS compared to the classical methods.

It is highly recommended to follow up this methodology by converting the manual calculations logic to a computerized package that can be used to solve transportation problems in any organization and also can be used in all books related to operations research and quantitative methods.

References

Anderson, Sweeney, Williams, Wisniewski. (2009). "An Introduction to Management Science". South-Western Cengage Learning- UK- P.306

Charnes A. and W. W. Cooper. (1954). "The stepping-stone method for explaining linear programming calculations in transportation problems". *Mgmt Sci.*1, 49-69. http://dx.doi.org/10.1287/mnsc.1.1.49

Dantzig .G.B. (1951). *"Application of the simplex method to a transportation problem. In Activity Analysis of Production and Allocation"* (T. C.KOOPMANS, Ed.), Chap. XXIII. Wiley, New York.

Dantzig G.B. (1963). *"Linear Programming and Extensions".* Princeton University Press, Princeton, NJ.

Edu, Bernard Enya, Esang, Atim E. & Otonkue, Agba D. O. (2009). Linear Programming: A Practical Approach to Transportation Cost Problems, *Social Science Research Network (SSRN)* -1434823.

Glover, F., D. Karney, D. Klingman, & A. Napier. (1974). "A Computation Study on Start Procedures, Basis Change Criteria and Solution Algorithms for Transportation Problems". *Management science journal,*20 , 5, pp793-819. http://dx.doi.org/10.1287/mnsc.20.5.783

Hamdy A Taha, Prentice Hall. (2002). *"Operations Research: An introduction"* 6 th Edition, p.165.

Hamdy A Taha. (2003). *"TORA Computer Algorithm for Operations Research. An Introduction"*, 7th ed 2003. Prentice Hall Inc.

Jay Heizer, Barry Render, Pearson Prentice Hall. (2004). *"Operations Management"* 7 Edition, P.688

Kirca O. & Satir A. (1990). "A heuristic for obtaining an initial solution for the transportation problem". *J. Opl Res.Soc.* 41, 865-871.

Ramakrishna C.S. (1988). "An Improvement to Goyal's Modified VAM for the Unbalanced Transportation Problem" *J. Opl Res.Soc.* Vol.39, No.6, pp. 609-610. http://dx.doi.org/10.1057/palgrave.jors.0390611

Appendix -1 – Details of calculations for the new methodology for (10x10) problem.

10x10	New Methodology IBFS Cost =62500									
	1000	500	200	300	300	600	100	500	400	200
500	10 350	30	50	70	20 150	40	20	35	40	15
300	20	30	35	45	65	55	25	15 300	10	50
700	40	10 150	20 200	15 300	30	25 50	45	55	60	25
250	60	15 50	30	20	60	45	35	40	25	10 200
750	20 50	20 300	35	25	55	40	30	50	20 400	30
700	10 600	25	25	30	40	35	25 100	45	30	40
500	20	30	15	40	35	15 500	20	40	10	45
100	25	35	10	50	25	20 50	15	20 50	25	55
150	30	45	25	35	15 150	30	10	30	30	30
150	55	30	45	15	20	10	20	15 150	25	15

Supply	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th	16th	17th
500	32.5	32.5	32.5	30	30	30	25	25	30	-	-	-	-	-	-	-	-
300	32.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
700	27.5	27.5	27.5	25	25	25	27.5	27.5	25	25	22.5	22.5	20	20	15	12.5	12.5
250	32.5	32.5	32.5	30	30	30	27.5	27.5	25	25	22.5	22.5	20	17.5	20	17.5	17.5
750	30	30	30	30	30	30	27.5	27.5	25	25	22.5	22.5	25	22.5	20	22.5	-
700	30	30	30	30	30	30	27.5	27.5	25	25	27.5	-	-	-	-	-	-
500	25	25	25	20	-	-	-	-	-	-	-	-	-	-	-	-	-
100	25	25	25	25	25	-	-	-	-	-	-	-	-	-	-	-	-
150	30	30	30	30	30	30	30	-	-	-	-	-	-	-	-	-	-
150	20	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

	1000	500	200	300	300	600	100	500	400	200
1st	22.5	30	27.5	32.5	32.5	32.5	22.5	37.5	25	30
2nd	25	30	25	30	30	30	20	40	25	30
3rd	22.5	27.5	25	32.5	32.5	32.5	22.5	40	27.5	30
4th	22.5	27.5	25	32.5	32.5	32.5	22.5	-	27.5	30
5th	25	25	25	30	30	35	25	-	30	30
6th	25	22.5	27.5	27.5	25	37.5	27.5	-	30	27.5
7th	25	22.5	27.5	27.5	35	-	27.5	-	30	27.5
8th	20	20	30	25	40	-	30	-	30	25
9th	20	20	30	25	-	-	30	-	30	25
10th	30	17.5	27.5	22.5	-	-	32.5	-	27.5	27.5
11th	30	17.5	27.5	22.5	-	-	-	-	27.5	27.5
12th	40	15	30	20	-	-	-	-	25	25
13th	-	15	30	0	-	-	-	-	25	25
14th	-	15	-	20	-	-	-	-	25	25
15th	-	15	-	20	-	-	-	-	25	-
16th	-	15	-	20	-	-	-	-	-	-
17th	-	12.5	-	17.5	-	-	-	-	-	-