Dependence Structure and Hedging of U.S. Spot and Futures Markets in Financial Crisis

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Abstract

The main objective of this study is to measure appropriately the dependence structure and optimal hedge ratio of U.S. spot and futures markets in financial crisis. In much empirical literature it has been demonstrated that linear Pearson correlation is not an appropriate dependence measure for non-normal distributions. This inadequacy of correlation requires an appropriate dependence measure: the copula. Copula modeling has become an increasingly popular tool in finance to model assets returns dependency as it can overcome the limitations of correlation when extreme losses occurred. The contribution of this paper is in two aspects. First, an appropriate copula function is discovered to capture the dependence structure of S&P 500 spot and futures in financial crisis adequately. Second, Gumbel copula function is exploited, with threshold GARCH model as marginals, to construct a Gumbel copula-threshold-GARCH model to estimate the optimal hedge ratio, simultaneously capturing asymmetric nonlinear behaviour in univariate returns of spot and futures markets and bivariate dependency.

Keywords: Dependence, Hedge ratio, Copula, Threshold-GARCH, Financial crisis

1. Introduction

The introduction of Stock index futures contracts is one of the significant financial innovations in the twentieth century. Institutional investors use stock index futures as a major risk managing tool. The transference of risk is the main functions of stock index futures markets. Stock index futures can be used to hedge market risk caused by spot price fluctuations. The key issue of futures market is to control or reduce the risk of the portfolio. Therefore, the hedger has to determine the optimal hedge ratio and improve the hedging effect to averse the price risk of spot market.

Many scholars have made their contribution to the optimal hedging ratio model development. One of the most widely-used hedging strategies is based on the minimization of the variance of the hedged portfolio. Edrington (1979) firstly applies the concept of portfolio theory to hedging, by using the ordinary least squares (OLS) regression model to estimate the optimal hedge ratio. Factors that influence the hedge construction and its effectiveness include basis risk, hedging horizon, and correlation between changes in the futures price and the cash price. When it comes to estimating the hedge ratios, many different techniques are currently being employed, ranging from simple to complex ones. Ghosh (1993) finds that there is a cointegration relationship between spot and futures price series, and the traditional OLS estimator is biased. Considering the co-integration relationship, Kroner and Sultan (1993) use constant conditional correlation CCC-GARCH model to estimate the risk-minimizing futures hedge ratios. Engle (2002) proposes a new class of multivariate models called dynamic conditional correlation (DCC) models. Ku, Chen & Chen (2007) apply the DCC model with error correction terms to investigate the optimal hedge ratios. Results show that the dynamic conditional correlation model yields the best hedging performance in futures markets. Following estimation of OLS, VECM, VECM-GARCH models, Kavussanos and Visvikis (2008) conclude that time-varying hedge ratios outperform alternative specifications in reducing market risk. Park and Jei (2010) extend Engle's dynamic conditional correlation (DCC) model to more flexible ones to analyze the optimal hedge ratio. They find that asymmetric and flexible density specifications help increase the goodness-of-fit of the estimated models, but do not guarantee higher hedging performance. Wei, Wang & Huang (2011) propose a new hedging model combining the newly introduced multifractal volatility (MFV) model and the dynamic copula functions. Using high-frequency intraday quotes of the spot Shanghai Stock Exchange Composite Index (SSEC), spot China

Securities Index 300 (CSI 300), and CSI 300 index futures, they compare the direct and cross hedging effectiveness of the copula–MFV model with several popular copula–GARCH models. The finding of this paper indicates that multifractal analysis may offer a new way of quantitative hedging model design using financial futures. Hou & Li (2013) employ multiple hedging models to calculate optimal hedge ratios and assess hedging performance of Chinese stock index futures. They find that the CSI 300 stock index futures can be an effective hedging tool and the question whether time-varying ratios outperform constant ratios depends on the length of the hedging horizon. Salvador and Aragó (2014) use nonlinear GARCH models to estimate optimal hedge ratios with futures contracts and obtain more efficient hedge ratios and superior hedging performance compared to the other methodologies.

However, most of these dynamic hedging models assume that the spot and futures returns follow a multivariate normal distribution with linear dependence. A popular used dependence measure is correlation, which indicates the strength and direction of a linear relationship between two random variables. The best known correlation measure is the Pearson correlation coefficient. It is a reasonable measure when the random variables are normally distributed. But much research shows that the multivariate normal distribution is inadequate because it underestimates both tail thickness of the marginal distributions and their dependence structure. Especially in financial crisis, data are approximated with more skewed distributions because of occasional, extreme losses. In 2008 financial crisis, U.S. stock markets have suffered their worst volatile trading days in memory, and various stock indices have fallen dramatically. The Dow Jones and S&P 500 are on course to record their worst yearly returns since the Great Depression. Meanwhile, U.S. stock index futures fluctuated a day after the Dow Jones snapped a seven-day losing streak. There is a great number of empirical evidence that the dependence between many important asset returns is non-normal in such crisis. Pearson correlation coefficient is not an appropriate dependence measure for very fat-tailed risks when extreme losses occurred. This inadequacy of correlation requires an appropriate dependence measure.

Copula method is the right tool which is applied to research on non-normal dependence of financial time series. Copula modeling has become an increasingly popular tool in finance to model assets returns dependency. In essence, copula functions enable us to extract the dependence structure from the joint distribution function of a set of random variables and, at the same time, to isolate such dependence structure from the univariate marginal behavior. Since Longin and Solnik (2001) have shown that the correlation between market returns is higher in case of extreme events, the number of papers on copula theory in finance and economics has grown enormously. Schmidt (2002) discusses the tail dependence property for some well-known examples of elliptical distributions. Breymann, Dias & Embrechts (2003) analyze the dependence structure within FX return data and conclude that the empirical fit of the t copula is often superior to Gaussian copula. Cherubini, Luciano & Vecchiato (2004) focus primarily on applications of copulas in mathematical finance and derivatives pricing. Demarta and McNeil (2005) present some new extensions of the t copula that follow from the representation of the multivariate t distribution relating extreme value theory. Nelsen (2006) provides detailed and readable introductions to copulas and their statistical and mathematical foundations. Rodriguez (2007) models dependence with switching-parameter copulas to study financial contagion. Sun, Rachev, Stoyanov & Fabozzi (2008) point out that when analyzing stock market, two dependence structures are encountered and copula is an alternative measure that can overcome the limitations of correlation. Bouye and Salmon (2009) introduce a general approach to nonlinear regression model based on the copula function that defines the dependency structure between the variables. Guegan and Zhang (2010) propose a dynamic copula for measuring dependence in multivariate financial data. Yin, Chokethaworn & Chaiboonsri (2013) set up the dependence structure between CSI300 index and futures by Copula-ARMA-GARCH models and find out which copula can provide a better fit to the empirical data. The empirical results indicate that there is high degree of dependence between CSI300 index and futures. The asymmetric tail dependence description is better, and tail dependence is significantly high. Nguyen, Bhatti & Hayat (2014) propose to blend copula functions with Asymmetric GARCH models and apply the procedure on the All Ordinaries Index and its corresponding Share Price Index on future contracts in Australia. The copula's marginals by the AGARCH processes can differentiate between the impacts of positive and negative shocks on the returns volatility while taking the large kurtosis of the returns into account. The findings reveal that the two spot and future markets show a strong right tail dependence but no left tail dependence. Maitra & Dey (2014) model the dependence structures of India's and Asian natural rubber futures (derivatives) markets. Analysis shows that relatively a high degree of dependence has been observed between India's and China's markets in comparison to other markets. This paper sheds light on the dominant role of copulas for attaining the methodological congruence of dependence-structure modelling.

The primary motivation for this paper is as follows. First, Copula models for financial time series are used to extract the dependence structure of spot and futures markets when crisis breaks out. Second, the optimal hedge ratio is estimated using Gumbel copula-threshold-GARCH model. A detailed analysis is available and the rest contents proceed as follows. Section 2 provides the methodology of risk-minimizing hedge theory, Copula functions and Copula--threshold-GARCH Hedging Model. Section 3 presents an elaborate discussion on the evidences from S&P 500 spot and futures markets, including illustrations of the empirical results. Section 4 concludes that the dependence between the return series of S&P 500 stock index and futures in 2008 financial crisis can be well captured by the Gumbel copula function. In line with recent research, the choice of a proper copula is significant to an accurate estimation of tail dependence in crisis. The Gumbel copula-threshold-GARCH model is employed to estimate the optimal hedge ratio. The methodology is quite flexible to model the dependence between two markets and estimate the optimal hedge ratio.

2. Methodology

2.1 Risk-minimizing hedge ratio

The main aim of hedging is assumed to be the minimization of the variance of the return on the portfolio. The optimal hedge ratio is defined as the ratio of futures holdings to a spot position that minimizes the risk of the hedged portfolio. Ederington derives hedge ratios that minimize the variance of the hedged portfolio, based on portfolio theory. Let ΔS_t and ΔF_t represent the log returns on spot and futures on the portfolio between period t and t-1. Define *h* as the ratio of futures holdings to a spot position that minimizes the risk of the hedged portfolio. Then R_t is the return on the portfolio, which is given by

$$R_t = \Delta S_t - h\Delta F_t \tag{1}$$

$$\Delta S_t = \ln S_t - \ln S_{t-1}, \quad t = 1,...,T \tag{2}$$

$$\Delta F_t = \ln F_t - \ln F_{t-1}, \quad t = 1,...,T \tag{3}$$

The expected return E (R_t) and variance (R_t) of the portfolio are respectively as follows.

$$E(R_t) = E(\Delta S_t) - hE(\Delta F_t) \tag{4}$$

$$\begin{aligned} Var(R_t) &= Var(\Delta S_t) - 2hCov(\Delta S_t, \Delta F_t) + h^2 Var(\Delta F_t) \\ &= \sigma^2_{\Delta S} - 2h\sigma_{\Delta S \Delta F} + h^2\sigma^2_{\Delta F} \end{aligned} \tag{5}$$

Where $\sigma_{\triangle S}$ is the standard deviation of $\triangle S_t$, $\sigma_{\triangle F}$ is the standard deviation of $\triangle F_t$, $\sigma_{\triangle S\triangle F}$ is the covariance of $\triangle S_t$ and $\triangle F_t$. The optimal hedging ratio should be the optimal solution of the unconstrained optimization problems.

$$\min Var(R_t) = \sigma^2_{\Delta S} - 2h\sigma_{\Delta S \Delta F} + h^2\sigma^2_{\Delta F} \tag{6}$$

If the joint distribution of spot and futures returns remains the same over time, then the conventional risk-minimizing hedge ratio h^* will be:

$$h^* = \frac{\mathrm{cov}(\Delta S, \Delta F)}{\mathrm{var}(\Delta F)} = \frac{\sigma_{\Delta S \Delta F}}{\sigma^2_{\Delta F}} = \frac{\rho\sigma_{\Delta S}\sigma_{\Delta F}}{\sigma^2_{\Delta F}} = \rho\frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} \tag{7}$$

Where ρ is the correlation coefficient between $\triangle S_t$ and $\triangle F_t$. Obviously, the dynamic hedge ratio depends on the way in which the condition variances and covariances are specified.

2.2 Copula functions

The correlation coefficient indicates the strength and direction of a linear relationship between two random variables. Although correlation plays a central role in finance, using linear correlation to measure the dependence of two markets may be misleading because it does not completely characterize the relationship between two variables, when distributions of return series are founded to be non-normality. It is not an appropriate dependence measure for leptokurtosis and fat-tailed distributions.

As is known to us, non-normality at the univariate level is associated with leptokurtosis phenomena, and the fat-tail problem. In a multivariate setting, the fat-tail problem can be referred both to the marginal univariate distributions and to the joint probability of large market movements. This concept is called tail dependence. The use of copula functions enables us to model these two features, fat tails and tail dependence, separately.

A copula function links *n* univariate marginal distributions to a full multivariate distribution resulting in a joint distribution function of *n* standard uniform random variables. Consider a vector random variable, $X = [X_1, X_2, \ldots, X_n]'$, with joint distribution F and marginal distributions $F_1, F_2, \ldots, F_n$. Sklar's (1959) theorem provides the mapping from the individual distribution functions to the joint distribution function:

$$F(\mathrm{x}) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)), \quad \forall \mathrm{x} \in \mathbb{R}^n. \tag{8}$$

From any multivariate distribution F, we can extract the marginal distributions F_i, and the copula C, which captures the dependency structure among $X_1, X_2, \ldots, X_n$. And, more useful for time series model, given any set of marginal distributions $(F_1, F_2, \ldots, F_n)$ and any copula C, equation (8) can be used to obtain a joint distribution with the given marginal distributions. The density function of $X = [X_1, X_2, \ldots, X_n]'$ in turn can be expressed in terms of the density copula and the marginal densities:

$$\begin{aligned} f(\mathrm{x}) &= \left(\frac{\partial^n C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))}{\partial F_1(x_1) F_2(x_2) \ldots F_n(x_n)} \right) \frac{\partial F_1(x_1)}{\partial x_1} \frac{\partial F_2(x_2)}{\partial x_2} \cdots \frac{\partial F_n(x_n)}{\partial x_n} \\ &= c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) \prod_{i=1}^{n} f_i(x_i) \end{aligned} \tag{9}$$

We now describe the copula functions of two random variables X and Y used in our empirical application. Then the standard formulation is:

$$H(x, y) = C(F(x), G(y)) \tag{10}$$

where $C(u,v)$ is the copula, F and G are marginal distribution functions, and H is the joint cumulative distribution function. The information of the marginal distributions are retained in $F(x)$ and $G(y)$, and the dependence information is summarized by $C(u,v)$. The dependence relationship is entirely determined by the copula, while the scaling and the shape (e.g., the mean, standard deviation, skewness and kurtosis) are entirely determined by the marginal. An important feature of this result is that the marginal distributions do not need to be in any way similar to each other, nor is the choice of copula constrained by the choice of marginal distributions. This flexibility makes copulas a potentially useful tool for building econometric models.

2.2.1 Elliptical copulas

Copulas can be distinguished in the Elliptical and Archimedean family. Elliptical copulas are the ones with elliptical distributions and therefore symmetry in the tails. Two frequently used copulas in this family are the Gaussian and the student's t copula.

(1) Gaussian copula

Assume there are two random variables X and Y, the Gaussian copula is defined by

$$\begin{aligned} C(u, v; \rho) &= \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v)) \\ &= \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}} dx\, dy \end{aligned} \tag{11}$$

where Φ_ρ is the bivariate standardized Gaussian cumulative distribution function (cdf) and the letter Φ represents the univariate standardized Gaussian cdf.

(2) Student's t copula

For the bivariate case, the Student's t copula is defined as

$$\begin{aligned} C(u, v; \rho, \upsilon) &= t_{\rho,\upsilon}(t_\upsilon^{-1}(u), t_\upsilon^{-1}(v)) \\ &= \int_{-\infty}^{t_\upsilon^{-1}(u)} \int_{-\infty}^{t_\upsilon^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \left(1 + \frac{x^2 - 2\rho xy + y^2}{\upsilon(1-\rho^2)}\right)^{-\frac{\upsilon+2}{2}} dxdy \end{aligned} \tag{12}$$

where $t_\upsilon(x) = \int_{-\infty}^{x} \frac{\Gamma((\upsilon+1)/2)}{\sqrt{\pi\upsilon}\Gamma(\upsilon/2)} \left(1 + \frac{z^2}{\upsilon}\right)^{-\frac{\upsilon+1}{2}} dz$.

2.2.2 Archimedean copulas

In comparison to Elliptical copulas, Archimedean copulas are constructed using a generator $\Phi_\alpha(t)$, indexed by the parameter α. By choosing the generator, one obtains a family of Archimedean copulas. The formulas of some well-known copulas for the bivariate cases are given as follows.

(1) Gumbel copula

$$C(u,v;\alpha)=\exp\left\{-[(-\ln u)^{\alpha}+(-\ln v)^{\alpha}]^{1/\alpha}\right\} \tag{13}$$

(2) Clayton copula

$$C(u,v;\alpha)=\max\left[(u^{-\alpha}+v^{-\alpha}-1)^{-1/\alpha},0\right] \tag{14}$$

(3) Frank copula

$$C(u,v;\alpha)=-\frac{1}{\alpha}\ln\left(1+\frac{(e^{-\alpha u}-1)(e^{-\alpha v}-1)}{e^{-\alpha}-1}\right) \tag{15}$$

2.3 Copula--threshold-GARCH Hedging Model

An appropriate way to capture the time varying nature of volatility is to model the conditional variance as a GARCH process. Firstly introduced by Engle (1982), Autoregressive Conditional Heteroskedasticity (ARCH) model is used to measure and forecast volatility of financial markets. However, in an ARCH (p) model, old news which arrived at the market more than p periods ago has no effect at all on current volatility. Furthermore, in many empirical applications with the linear ARCH (p) model a relatively long lag length in the conditional variance equation is often called for. In this light, the Generalized ARCH, or GARCH (p, q) model allowing for both a longer memory and more flexible lag structure is advanced by Bollerslev (1986). The GARCH (p, q) process is then given by

$$r_t=u_t+\varepsilon_t\,,\qquad \varepsilon_t|\psi_{t-1}\sim \mathrm{N}(0,\sigma_t^2) \tag{16}$$

$$\sigma_t^2=\alpha_0+\sum_{i=1}^{p}\alpha_i\varepsilon_{t-i}^2+\sum_{j=1}^{q}\beta_j\sigma_{t-j}^2 \tag{17}$$

Where ε_t denotes a real-valued discrete-time stochastic process, Ψ_{t-1} is the information set generated by the past values of ε_t, σ_t^2 is known as the conditional variance since it is a multi-period ahead estimate for the variance calculated based on any past relevant information.

Using the GARCH model, it is possible to interpret the current fitted variance h_t. Following the idea of Bollerslev, Engle and Jeffrey (1988), p=q=1 is found to suffice in most applications. Then, GARCH (1, 1) formula is

$$\sigma_t^2=\alpha_0+\alpha_1\varepsilon_{t-1}^2+\theta_1\sigma_{t-1}^2 \tag{18}$$

It is widely applied to empirical studies as it can capture important characteristics of the high frequency time series data, as described by Cont and Fonseca (2001, 2002). The most interesting feature not addressed by this model is asymmetric effect confirmed by French Schwert and Stambaugh (1987), Nelson(1991). This effect occurs when a negative shock (bad news) to financial time series is likely to cause volatility to rise by more than a positive shock (good news) of the same magnitude.

Since the GARCH models are incapable of separating out the asymmetric information, Glosten developed a model that allows the effects of good and bad news to have different effects on volatility. The threshold-GARCH model is a simple extension of GARCH with an additional term added to account for possible asymmetries. The conditional variance is expressed in this form:

$$\sigma_t^2=\alpha_0+\alpha_1\varepsilon_{t-1}^2+\gamma \mathrm{I}_{t-1}\varepsilon_{t-1}^2+\theta_1\sigma_{t-1}^2 \tag{19}$$

Where I_{t-1}=1 if ε_t<0, I_{t-1}=0 otherwise. For a leverage effect, we would see γ>0.

The conditional variances of changes in the spot and futures prices at time t are as follows.

$$\begin{aligned}\sigma_{\Delta S,t}^2&=\alpha_{\Delta S,0}+\alpha_{\Delta S,1}\varepsilon_{\Delta S,t-1}^2+\gamma_{\Delta S}\mathrm{I}_{t-1}\varepsilon_{t-1}^2+\beta_{\Delta S,1}\sigma_{\Delta S,t-1}^2\\ \sigma_{\Delta \mathrm{F},t}^2&=\alpha_{\Delta \mathrm{F},0}+\alpha_{\Delta \mathrm{F},1}\varepsilon_{\Delta \mathrm{F},t-1}^2+\gamma_{\Delta \mathrm{F}}\mathrm{I}_{t-1}\varepsilon_{t-1}^2+\beta_{\Delta \mathrm{F},1}\sigma_{\Delta \mathrm{F},t-1}^2\end{aligned} \tag{20}$$

Then, then the conventional risk-minimizing hedge ratio will be:

$$h^* = \frac{\text{cov}(\Delta S, \Delta F)}{\text{var}(\Delta F)} = \frac{\sigma_{\Delta S \Delta F,t}}{\sigma^2_{\Delta F,t}} = \frac{\rho_{\Delta S \Delta F}\sigma_{\Delta S,t}\sigma_{\Delta F,t}}{\sigma^2_{\Delta F,t}} = \rho_{\Delta S \Delta F}\frac{\sigma_{\Delta S,t}}{\sigma_{\Delta F,t}} \tag{21}$$

Where h* is the optimal hedge ratio, and $\rho_{\Delta S \Delta F}$ is the correlation coefficient of spot and futures markets.

The conventional risk-minimizing hedge ratio mentioned in the previous section is estimated under the assumption of multivariate normality. By contrast, the use of a copula function allows us to consider the marginal distributions and the dependence structure both separately and simultaneously. Therefore, the joint distribution of the asset returns can be specified with full flexibility, which will thus be more realistic.

$$h^* = \rho^*_{\text{Copula}}\frac{\sigma_{\Delta S,t}}{\sigma_{\Delta F,t}} \tag{22}$$

where ρ^*_{Copula} is the median correlation coefficient of copula function instead of conventional linear Pearson correlation coefficient. According to the definition and mathematical properties of copula function, given u=v=50%, the median correlation coefficient is

$$\rho^*_{\text{Copula}} = 4C(50\%, 50\%) - 1 \tag{23}$$

This paper use the copula to calculate the median correlation parameter to matching the spot and futures return rate nonlinearly, so the dependence between the return time series of S&P 500 stock index and futures in extreme condition will be guaranteed and the hedge efficiency will be enhanced.

3. Data and empirical results

3.1 Data

We investigate the dependence between the S&P 500 stock index and futures time series data in U.S. financial crisis. On September 15th, 2008, bankruptcy of the investment bank “Lehman Brothers” in U.S. marked the beginning of global crisis. It resulted in a number of bank failures and sharp reductions in the value of stock worldwide. Then, a great many of the world’s stock exchanges experienced the worst declines in their history, with drops of around 10% in most indices. For this reason, the empirical data covers the period from September 15th, 2008 to July 31st, 2009 when the stock and futures markets suffered a dramatic fluctuation in the crisis.

The test data is the natural logarithm return of the closing price. All the estimation process is carried out in Matlab 7.7.0. Some descriptive statistics are presented in Table 1. As previously found in other studies, returns exhibit excess kurtosis and skewness. The value of Jarque-Bera statistic refuses the assumption of normal distribution. It is also illustrated in Figure 1. Distributions of return series are founded to be non-normality. As we can not make sure the population distributions of S&P 500 stock index and futures return series easily, the sample empirical distribution functions and kernel distribution estimates were shown in Figure 2.

Table 1. Descriptive statistics of S&P 500 stock index and futures returns series

Series	*Mean*	*Std. Dev.*	*Skewness*	*Kurtosis*	*Jarque-Bera*	*Probability*
S&P 500	-0.0011	0.0300	-0.0483	4.6509	140.8539	0.0000
S&P 500 futures	-0.0011	0.0303	0.1808	5.7877	295.1440	0.0000

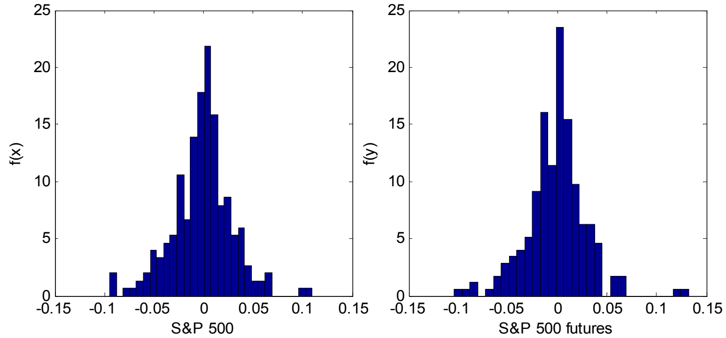

Figure 1. Frequency histograms of S&P 500 and S&P 500 futures returns series

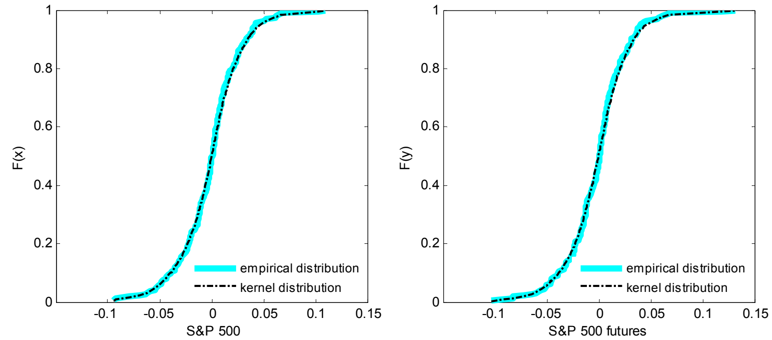

Figure 2. The empirical distribution and kernel distribution of S&P 500 futures and S&P 500

3.2 Copula choice for S&P 500 stock index and futures

The parameters and formulas of Elliptical and Archimedean copulas were estimated as follows.

3.2.1 Elliptical copulas

The correlation coefficient ρ of Gaussian copula function is

$$\hat{\rho}_{Gaussian} = \begin{pmatrix} 1.0000 & 0.9843 \\ 0.9843 & 1.0000 \end{pmatrix} \quad (24)$$

The Gaussian copula function is

$$\hat{C}_{Gaussian}(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-0.9843^2}} e^{-\frac{x^2-2\times 0.9843xy+y^2}{2(1-0.9843^2)}} dxdy \quad (25)$$

The correlation coefficient ρ and the degrees of freedom of Student's t copula function are

$$\hat{\rho}_t = \begin{pmatrix} 1.0000 & 0.9871 \\ 0.9871 & 1.0000 \end{pmatrix}, \ \hat{k} = 2.9421 \approx 3 \quad (26)$$

The Student's t copula function is

$$\hat{C}_t(u,v) = \int_{-\infty}^{t_3^{-1}(u)} \int_{-\infty}^{t_3^{-1}(v)} \frac{1}{2\pi\sqrt{1-0.9871^2}} (1 + \frac{x^2 - 2\times 0.9871xy + y^2}{3\times(1-0.9871^2)})^{-\frac{3+2}{2}} dxdy \quad (27)$$

The density functions $c(u,v)$ of Gaussian and Student's t copula are plotted in Figure 3. It is illustrated that both these copulas have symmetric tails and therefore strong tail dependence exists between S&P 500 stock index and futures returns series. However, they have different characteristics in terms of tail dependence. The density function of Student's t copula has a little stronger tail than Gaussian copula.

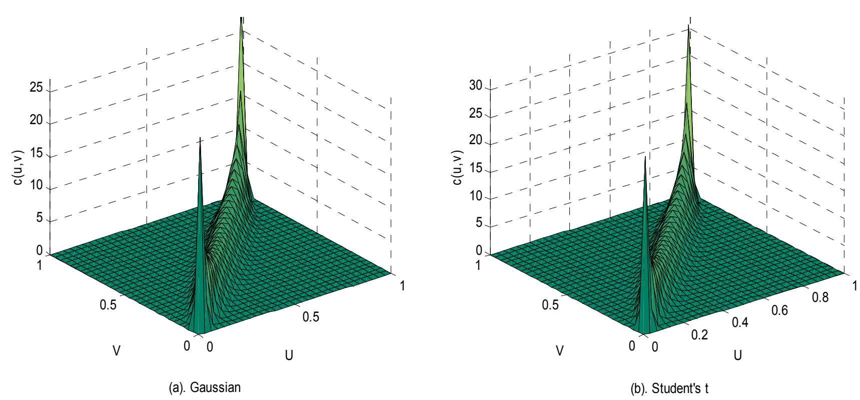

Figure 3. The density function c(u,v) of Gaussian and Student's t copula

3.2.2 Archimedean copulas

The parameters α of Archimedean copulas are estimated and the formulas of Gumbel copula, Clayton copula and Frank copula are followed in Table 2.

Table 2. Estimated results of Archimedean copulas

Archimedean copulas	*Estimated parameter α*	*Formulas of copulas*
Gumbel copula	9.1269	$\hat{C}_{Gumbel}(u,v) = \exp\left\{-[(-\ln u)^{9.1269} + (-\ln v)^{9.1269}]^{1/9.1269}\right\}$
Clayton copula	9.2345	$\hat{C}_{Clayton}(u,v) = \max\left[(u^{-9.2345} + v^{-9.2345} - 1)^{-1/9.2345}, 0\right]$
Frank copula	34.8682	$\hat{C}_{Frank}(u,v) = -\frac{1}{34.8682}\ln(1 + \frac{(e^{-34.8682u} - 1)(e^{-34.8682v} - 1)}{e^{-34.8682} - 1})$

The density functions $c(u,v)$ of Gumbel, Clayton and Frank copulas are plotted in Figure 4. As shown that, they have different characteristics in terms of tail dependence. The Gumbel copula has asymmetric tails and the upper tail is stronger. The Clayton copula also has asymmetric tails, but differently, the lower tail is stronger than upper. The lower left tails are best described with Clayton copulas while the upper right tails are best described with Gumbel copula. Different with the former two copulas, the density functions of Frank copula are symmetry in the tails.

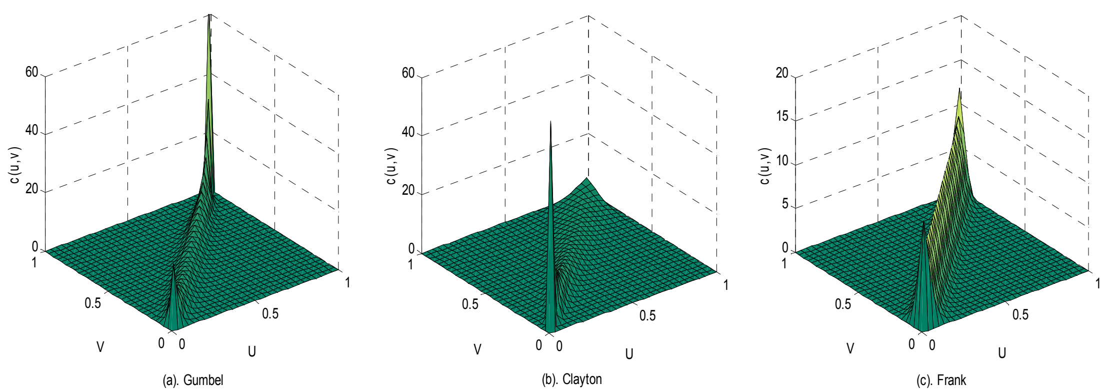

Figure 4. The density function c(u,v) of Gumbel, Clayton and Frank copula

3.2.3Comparison and evaluation

In order to choose an appropriate copula model to describe the dependence structure of data, we introduce empirical copula to evaluate performances of the Elliptical and Archimedean family. When analyzing data with an unknown underlying distribution, one can transform the empirical data distribution into an empirical copula by warping such that the marginal distributions become uniform. Mathematically the empirical copula frequency function is calculated by

$$\hat{C}_{Empirical}(u,v) = \frac{1}{n}\sum_{i=1}^{n} I_{[F_n(x_i)\le u]} I_{[G_n(y_i)\le v]}, \quad u,v \in [0,1] \tag{28}$$

where

$$I_{[F_n(x_i)\le u]} = \begin{cases} 1 & F_n(x_i) \le u \\ 0 & else \end{cases} \tag{29}$$

The Square of Euclidean distance between each copula function and empirical copula are computed as

$$d^2_{Gauss} = \sum_{i=1}^{n} \left|\hat{C}_{Empirical}(u_i,v_i) - \hat{C}_{Gauss}(u_i,v_i)\right|^2 = 0.0091 \tag{30}$$

$$d_t^2 = \sum_{i=1}^{n} \left| \hat{C}_{Empirical}(u_i, v_i) - \hat{C}_t(u_i, v_i) \right|^2 = 0.0086 \qquad (31)$$

$$d_{Gumbel}^2 = \sum_{i=1}^{n} \left| \hat{C}_{Empirical}(u_i, v_i) - \hat{C}_{Gumbel}(u_i, v_i) \right|^2 = 0.0078 \qquad (32)$$

$$d_{Clayton}^2 = \sum_{i=1}^{n} \left| \hat{C}_{Empirical}(u_i, v_i) - \hat{C}_{Clayton}(u_i, v_i) \right|^2 = 0.0864 \qquad (33)$$

$$d_{Frank}^2 = \sum_{i=1}^{n} \left| \hat{C}_{Empirical}(u_i, v_i) - \hat{C}_{Frank}(u_i, v_i) \right|^2 = 0.0170 \qquad (34)$$

By comparing the distances, we found that the distance between Gumbel copula and empirical copula is the smallest. It is suggested that the Gumbel copula can provide a better fit to the empirical data and therefore well extract the dependence structure between S&P 500 stock index and futures in financial crisis.

3.3 Estimating of hedge ratio

The selected copula is a Gumbel with parameter α = 9.1269. The median correlation coefficient of Gumbel copula function is

$$\rho_{\text{Copula}}^{*} = 4C(50\%, 50\%) - 1 = 4 * \exp\left\{-[(-\ln 0.5)^{9.1269} + (-\ln 0.5)^{9.1269}]^{1/9.1269}\right\} - 1 = 0.8936 \qquad (35)$$

The optimal hedge ratios estimated by Copula-based GARCH hedging model are dynamic time series, illustrated in Figure 5. The mean of hedge ratios is 0.9083, while the maximum is 1 and the minimum is 0.7561, as is shown in Figure 6. It has been demonstrated that the traditional static approach is inappropriate for hedging with futures, with the result that a variety of alternative dynamic hedging strategies has emerged. The optimal hedge ratio determined by the fluctuations of spot and futures prices is likely to change much through time.

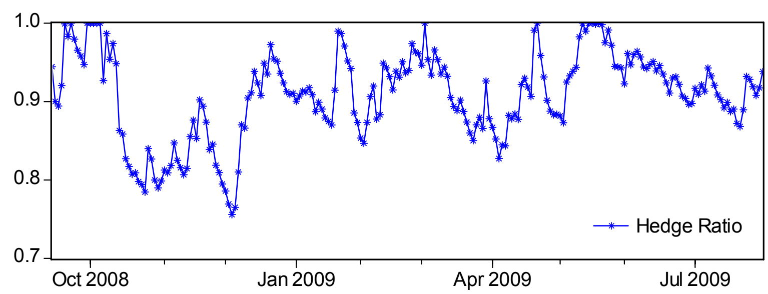

Figure 5. Dynamic hedge ratios estimated by Copula-based GARCH hedging model

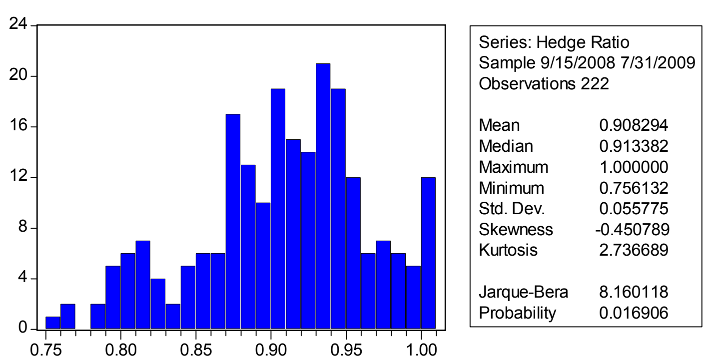

Figure 6. Some descriptive statistics of dynamic hedge ratios

4. Conclusions

In this study, we discussed the dependence structure and optimal hedge ratio of U.S. spot and futures markets in financial crisis. The optimal hedge ratio can differ significantly depending on the dependence structure of spot and futures markets. The superiority of the time-varying hedge ratio essentially comes from taking account of the changing joint distribution of spot and futures returns. Modeling the dependence structure of spot and futures markets is important to determine the optimal hedge ratio. However, most dynamic hedging models assume that the spot and futures returns follow a multivariate normal distribution with linear dependence. This assumption is at odds with numerous empirical studies, in which it has been shown that many financial asset returns are skewed, leptokurtic, and asymmetrically dependent. Copula modeling can overcome the limitations of correlation when extreme losses occurred.

The choice of an appropriate copula function is aimed at adequately capturing the dependence between S&P 500 spot and futures market in 2008 financial crisis. Data are approximated with more skewed distributions because of occasional, extreme losses. In this case, the Elliptical and Archimedean family of copulas are employed to extract the dependence structure. Among the Elliptical copulas, the Gaussian and the student's t copula show symmetric tails. However, the Archimedean copulas have different characteristics in terms of tail dependence. The Gumbel and Clayton copula have asymmetric tails, but differently, the former is stronger in the upper tail and the latter is stronger in the lower tail. Different with these two copulas, the density functions of Frank copula are symmetry in the tails. To choose an appropriate copula model to describe the dependence structure of data, we introduce empirical copula to evaluate performances of the Elliptical and Archimedean family. By comparing the distances between each copula function and empirical copula, we concluded that the dependence between the return series of S&P 500 stock index and futures in 2008 financial crisis can be well captured by the Gumbel copula function. In line with recent research, the choice of a proper copula is significant to an accurate estimation of tail dependence in crisis.

To estimate the optimal hedge ratio, a Gumbel copula-threshold-GARCH model is employed, simultaneously capturing asymmetric nonlinear behaviour in univariate returns of spot and futures markets and bivariate dependency. The methodology is quite flexible to model the dependence between two markets and estimate the optimal hedge ratio. As to the selection of a specific copula function in improving the hedging performance, more research is needed.

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References

Bouyé, E., & Salmona, M. (2009). Dynamic copula quantile regressions and tail area dynamic dependence in Forex markets. *The European Journal of Finance, 15*(7&8), 721-750. http://dx.doi.org/10.1080/13518470902853491

Breymann, W., Dias, A., Embrechts, P. (2003). Dependence structures for multivariate high-frequency data in finance. *Quantitative Finance*, 3(1), 1-14. http://dx.doi.org/10.1088/1469-7688/3/1/301

Cherubini, U., Luciano, E., & Vecchiato, W. (2004). *Copula Methods in Finance*. John Wiley & Sons Ltd. http://dx.doi.org/10.1002/9781118673331

Demarta, S., & McNeil A. J. (2005). The t copula and related copulas. I*nternational Statistical Review, 73*(1), 111-129. http://dx.doi.org/10.1111/j.1751-5823.2005.tb00254.x

Ederington, L. H. (1979). The hedging performance of the new futures markets. *Journal of Finance, 34*(1), 157-170. http://dx.doi.org/10.1111/j.1540-6261.1979.tb02077.x

Engle, R. (2002). Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics, 20*(3), 339-350. http://dx.doi.org/10.1198/0735001022886

Ghosh, A. (1993). Cointegration and error correction models: intertemporal causality between index and futures prices. *Journal of Futures Markets, 13*(2), 193-198. http://dx.doi.org/10.1002/fut.3990130206

Guegan, D., & Zhang, J. (2010). Change analysis of a dynamic copula for measuring dependence in multivariate financial data. *Quantitative Finance, 10*(4), 421-430. http://dx.doi.org/10.1080/14697680902933041

Hou, Y., & Li, S. (2013). Hedging performance of chinese stock index futures: an empirical analysis using wavelet analysis and flexible bivariate garch approaches. *Pacific-Basin Finance Journal, 24*(3), 109–131.

http://dx.doi.org/10.1016/j.pacfin.2013.04.001

Kavussanos, M. G., & Visvikis, I. D. (2008). Hedging effectiveness of the Athens stock index futures constracts. *The European Journal of Finance, 14*(3), 243-270. http://dx.doi.org/10.1080/13518470801890701

Kroner, K. F., & Sultan, J. (1993). Time-varying distributions and dynamic hedging with foreign currency futures. *The Journal of Financial and Quantitative Analysis, 28*(4), 535-551. http://dx.doi.org/10.2307/2331164

Ku, Y. H. H., Chen, H. C., & Chen K H. (2007). On the application of the dynamic conditional correlation model in estimating optimal time-varying hedge ratios. *Applied Economics Letters, 14*(7), 503 – 509. http://dx.doi.org/10.1080/13504850500447331

Longin, F., & Solnik, B. (2001). Extreme correlation of international equity markets. *The Journal of Finance, 56*(2), 649-676. http://dx.doi.org/10.1111/0022-1082.00340

Maitra, D., & Dey, K. (2014). Copulas and dependence structures: evidences from india's and asian rubber futures markets. International *Journal of Financial Markets & Derivatives, 3*(4), 322-357. http://dx.doi.org/10.1504/IJFMD.2014.062380

Nelsen, R. B. (2006). *An Introduction to Copulas* (2nd ed.). New York, NY: Springer.

Nguyen, C., Bhatti, M. I., & Hayat, A. (2014). Volatility linkages in the spot and futures market in australia: a copula approach. *Quality & Quantity, 48*(5), 2589-2603. http://dx.doi.org/10.1007/s11135-013-9909-2

Park, S. Y., & Jei, S. Y. (2010). Estimation and hedging effectiveness of time-varying hedge ratio: flexible bivariate GARCH approaches. *The Journal of Futures Markets, 30*(1), 71-79. http://dx.doi.org/10.1002/fut.20401

Rodriguez, J. C. (2007). Measuring financial contagion: a copula approach. *Journal of Empirical Finance, 14*(3), 401-423. http://dx.doi.org/10.1016/j.jempfin.2006.07.002

Salvador, E., & Aragó, V. (2014). Measuring hedging effectiveness of index futures contracts: Do dynamic models outperform static models? A regime-switching approach. *Journal of Futures Markets, 34*(4), 374–398. http://dx.doi.org/10.1002/fut.21598

Schmidt, R. (2002). Tail dependence for elliptically contoured distributions. *Mathematical Methods of Operations Research, 55*(2), 301-327. http://dx.doi.org/10.1007/s001860200191

Sun, W., Rachev, S., Stoyanov, S. V., Fabozzi, F. J. (2008). Multivariate skewed Student's t copula in the analysis of nonlinear and asymmetric dependence in the German equity market. *Studies in Nonlinear Dynamics & Econometrics, 12*(2), 1-32. http://dx.doi.org/10.2202/1558-3708.1572

Wei, Y., Wang, Y., & Huang, D. (2011). A copula–multifractal volatility hedging model for csi 300 index futures. Physica *A Statistical Mechanics & Its Applications, 390*(23), 4260-4272. http://10.1016/j.physa.2011.06.042