Optimal Retirement Age and Lower Bound on Interest Rate

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Received: November 20, 2023 Accepted: December 19, 2023 Online Published: December 26, 2023

doi:10.5430/afr.v13n1p1 URL: https://doi.org/10.5430/afr.v13n1p1

Abstract

For a long time now, the monetary policies in the US and other industrialized countries have been a regime of cheap money with the interest rate often being negative. A low interest rate in the framework of the Keynesian monetary policy stimulates demand but is hard on the retirees who are mostly on a fixed income. This paper generates a simple model to understand the interaction between current wealth, savings rate, interest rate and retirement age. The model also suggests a lower bound on the interest rate. The minimum interest rate at which Americans of median age with median wealth and median income, and retiring at the age of 67, turns out to be 10%.

Keywords: retirement age, Lower Bound on Interest

1. Introduction

The average retirement age in America is 65 for men and 63 for women. However most people can collect the full amount of Social Security at the age of 67. We also know that for years, the monetary policies in the US and other industrialized countries have been a regime of cheap money with the interest rate often being negative. A low interest rate in the framework of the Keynesian monetary policy stimulates demand but is hard on the retirees who are mostly on a fixed income. This paper generates a simple model to understand the interaction between current wealth, savings rate, interest rate and retirement age. The model also suggests a lower bound on the interest rate. The minimum interest rate at which Americans of median age with median wealth and median income, and retiring at the age of 67, turns out to be 10%.

The paper is organized as follows. The next section reviews the literature to provide the context for this paper. In the section titled A Simple Model Of Retirement Age I develop the model. The next section develops the Lower Bound on Interest Rate. I then examine the impact of accumulated wealth, yearly savings and interest rate on the minimum retirement age. The last section concludes.

2. Literature Review

There is a wealth of literature investigating the finances during retirement. Helen (2015) finds that medical costs are a significant determinant of hardship in retirement. Hosseini (2015) finds that mandatory annuitization in the Social Security results in a crowding out effect and the welfare gain is thus small. Hou and Sanzenbacher (2021) find that Black and Hispanic households have less than half the retirement wealth of White households and this discrepancy would have been higher but for Social Security. The rise in retirement wealth inequality is also corroborated by Sebastian et al. (2016). Seligman and Holden (2019) investigate how retirees manage their retirement wealth.

2.1 A Simple Model of Retirement Age

Let us consider a simple model where a representative agent aged A is considering an appropriate age for their retirement.

2.1.1 Notation

A Current age of the economic agent.

R Retirement age of the economic agent.

D Age of Death of the economic agent.

W Wealth of the economic agent at age A.

S Savings each year of the economic agent from age A + 1.

$-X$ Expenses each year of the economic agent during retirement.

i Interest rate.

Time	A	A+1	A+2	…	…	R	R+1	R+2	…	…	D
Time Rescaled	0	1	2	…	…	R-A	R-A+1	R-A+2	…	…	D-A
Cash Flow	W	S	S	…	…	S	-X	-X	…	…	-X

For successful retirement, NPV of the above cash flow should be greater than or equal to zero.

$$W + S\frac{(1+i)^{R-A}-1}{i(1+i)^{R-A}} - X\frac{(1+i)^{D-R}-1}{i(1+i)^{D-R}(1+i)^{R-A}} \geq 0$$

$$\Longrightarrow \quad Wi(1+i)^{D-R}(1+i)^{R-A} + S(1+i)^{D-R}[(1+i)^{R-A}-1] - X[(1+i)^{D-R}-1] \geq 0$$

$$\Longrightarrow \quad Wi(1+i)^{D-A} + S(1+i)^{D-A} - S(1+i)^{D-R} - X[(1+i)^{D-R}-1] \geq 0 \qquad (1)$$

From inequality (1) we can see that the inequality will be satisfied if

$$Wi(1+i)^{D-A} + S(1+i)^{D-A} - S(1+i)^{D-R} \geq X(1+i)^{D-R}$$

$$\Rightarrow \quad Wi(1+i)^{D-A} + S(1+i)^{D-A} \geq (S+X)(1+i)^{D-R}$$

$$\Rightarrow \quad (Wi+S)(1+i)^{R-A} \geq (S+X) \qquad (2)$$

Simplifying and taking ln of both sides, we get

$$R - A \geq \frac{1}{ln(1+i)}[ln(S+X) - ln(Wi+S)]$$

$$\Longrightarrow R \geq A + \frac{1}{ln(1+i)}[ln(S+X) - ln(Wi+S)] \qquad (3)$$

2.2 A Numerical Example

In order to understand the significance of (3), we generate numeric answers. For the purpose of this example, we assume the following:

A=38.3 years Median Age Source US Census,2019

W=$118,200 Median Wealth Source US Census,2019

Income=$67,520 Median Income Source US Census,2020

Savings Rate 5% Source: https://www.bea.gov/data/income- saving/personal-saving-rate

S=$3376=$67520*0.05

X=$54016 Expenditure during retirement is assumed to be at 80% of pre-retirement income. This is a standard assumption among financial advisors.

The minimum retirement age for different interest rates are given in Table 1.

Specifically, for this table W = 118, 200, A = 38.3, S = 3376, and X = 54016.

Table 1. Minimum Retirement Age For Different Interest Rates for US Median Data

Interest Rate	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%
Minimum Retirement Age (years)	293	155	110	88	76	68	62	58	55	52	50	49	47	46	45

2.3 Lower Bound on Interest Rate

We can also obtain a lower bound for interest rate.

From (2) $(Wi + S)(1 + i)^{R-A} \geq (S + X)$

By Expanding $(1 + i)^{R-A}$ and neglecting terms of second order and above

$$(Wi + S)[1 + i(R - A)] \geq (S + X)$$

$$\Rightarrow W(R - A)i^2 + [S(R - A) + W]i + S \geq (S + X)$$

$$\Rightarrow W(R - A)i^2 + [S(R - A) + W]i - X \geq 0 \quad (4)$$

$$\Longrightarrow i \geq \frac{-[S(R - A) + W] + \sqrt{[S(R - A) + W]^2 + 4W(R - A)X}}{2W(R - A)} \quad (5)$$

Using the figures used earlier along with the additional assumption that we want Americans to retire at the age of 67, we get the lower bound on i to be 10%. So for US macroeconomic policy the interest rate should be at least 10% using median values for various independent variables. In Figure 1 we can see the nature of the quadratic expression on the LHS of (4).

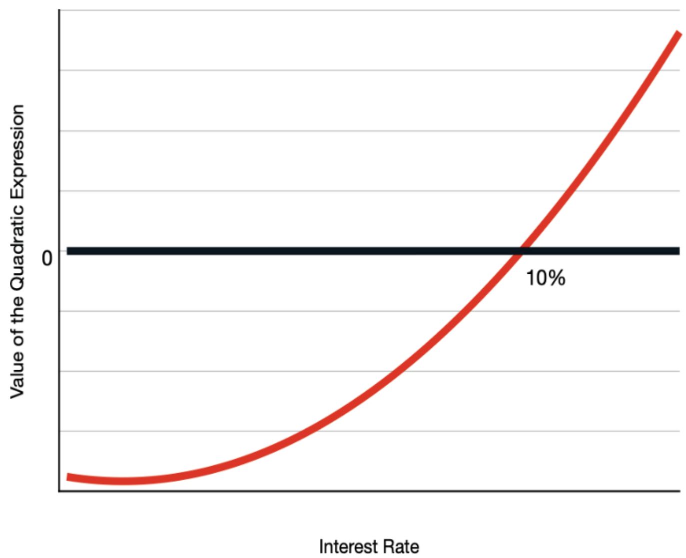

Figure 1. Value of Quadratic Expression with Interest Rate For US Median Data

This graph plots the quadratic expression on the LHS of (4) viz. $W(R - A)i^2 + [S(R - A) + W]i - X$ for various values of i. This graph is drawn for the values of parameters as outlined in section *2.2*. Specifically for this graph W = 118, 200, R = 67, A = 38.3, S = 3376, and X = 54016. As can be seen from the graph the quadratic expression is positive for $i \geq 10\%$

2.4 Comparative Statics

From (3), we get, $R_{min} = A + \frac{ln\frac{S+X}{Wi+S}}{ln(1+i)}$ where R_{min} is the Minimum Retirement Age. Therefore, taking partial derivatives with respect to W, S, X and i we get

$$\frac{\partial R_{min}}{\partial W} = \frac{\partial}{\partial W}\left[A + \frac{\ln\frac{S + X}{Wi + S}}{\ln(1 + i)}\right]$$

$$= \frac{\partial}{\partial W}\left[A + \frac{\ln(S + X)}{\ln(1 + i)} - \frac{\ln(Wi + S)}{\ln(1 + i)}\right]$$

$$= -\frac{\mathrm{i}}{(\mathrm{Wi}+\mathrm{S})\ln(1+\mathrm{i})} < 0 \quad \forall\, \mathrm{W} \geq 0 \tag{6}$$

⟹Higher the accumulated wealth, shorter is the age of retirement.

$$\frac{\partial R_{min}}{\partial S} = \frac{\partial}{\partial S}\left[A + \frac{ln(S+X)}{ln(1+i)} - \frac{ln(Wi+S)}{ln(1+i)}\right]$$

$$= \frac{1}{ln(1+i)}\left[\frac{1}{S+X} - \frac{1}{Wi+S}\right]$$

$$= \frac{1}{ln(1+i)}\left[\frac{Wi+S-S-X}{(S+X)(Wi+S)}\right]$$

$$= \frac{1}{ln(1+i)}\left[\frac{Wi-X}{(S+X)(Wi+S)}\right]$$

$$\therefore \frac{\partial R_{min}}{\partial S} \geq 0 \quad \forall Wi \geq X \tag{7}$$

⟹ If the retirement expenses can be financed out of the interest income on accumulated wealth, then the agent can retire right now and does not need to work anymore.

$$\& \ \frac{\partial R_{min}}{\partial S} \leq 0 \quad \forall Wi < X \tag{8}$$

⟹The agent needs to work till the retirement expenses can be financed out of the interest income on accumulated wealth.

$$\frac{\partial \mathrm{R_{min}}}{\partial \mathrm{i}} = \frac{\partial}{\partial \mathrm{i}}\left[\mathrm{A} + \frac{\ln(\mathrm{S}+\mathrm{X})}{\ln(1+\mathrm{i})} - \frac{\ln(\mathrm{Wi}+\mathrm{S})}{\ln(1+\mathrm{i})}\right]$$

$$= \frac{\partial}{\partial \mathrm{i}}\left[\mathrm{A} + \frac{\ln(\mathrm{S}+\mathrm{X}) - \ln(\mathrm{Wi}+\mathrm{S})}{\ln(1+\mathrm{i})}\right]$$

$$= \frac{1}{[ln(1+i)]^2}\left[-\frac{W ln(1+i)}{Wi+S} - \frac{ln(S+X) - ln(Wi+S)}{1+i}\right] \tag{9}$$

Inspecting (9) we see that the first term within the square bracket will dominate because the magnitude of W is much more than the rest of the terms and that logarithm is a contraction mapping.

$$\therefore \frac{\partial R_{min}}{\partial i} < 0 \tag{10}$$

⟹ Higher the interest rate lower will be the retirement age.

3. Conclusion

In this paper I have used a simple model to figure out the minimum retirement age. We derive Minimum Retirement Age (R_{min}) as a function of accumulated wealth, yearly saving and yearly expenditure in retirement years. From the comparative statics analysis, we see that the lower the interest rate the more the working years of the economic agent.

There is a welfare implication of this. If the Government of a country desires that the citizens should be able to retire at a specific age then they need to think of transfer payments to lower income segments if the interest rate is too low. The Keynesian theory of monetary policy assumes that the interest rate cannot be negative-an assumption which has been seen to be violated by deliberate policy in recent years (see Kirsch (2021), Bhansali (2020), Czudaj (2020), Dickler (2020), Lopez et al. (2020)). Monetary policy has real world impact through multiple channels- through housing decisions and decision to save for retirement (see Scholz et al. (2006), Case et al. (2013); for an interesting approach to forecasting see Guo (2021)). I am not aware of any theoretical development in Keynesian theory of monetary policy incorporating negative interest rates. That would be one of the possible extensions of future research efforts.

The other interesting finding is that of a lower bound on interest rates. I have arrived at a minimum interest rate for the US of 10% using the median figures for America.

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